Electricity Market-Clearing With Stochastic Security

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Abstract

In this dissertation we formulate a short-term electricity market-clearing problem with stochastic security criteria. The proposed stochastic security criteria make use of probabilistic measures of the expected load not served or of the loss-of-load probability associated with the random failures of pre-selected sets of generators, lines as well as load disturbances. We show that by economically penalizing the operation of the market through the associated demand-side costs of involuntary load shedding, the reserve service requirements are determined implicitly, thus removing the needs for specifying any *a priori* reserve requirements. Under this approach, the market-clearing problem gains in flexibility as it can balance the respective expected costs of: (i) the pre-contingency preventive security control actions that include unit commitment, generation and load dispatch as well as reserve scheduling; (ii) the post-contingency corrective actions that deploy reserves through further unit commitment decisions and load and generation re-dispatch; and, (iii) any post-contingency involuntary load shedding decisions. Case studies illustrate that electricity market-clearing with stochastic security leads to non-negligible economic savings for society, while it can still ensure that consumers benefit from a secure supply of electricity given how they value load shedding.

We derive theoretical results pertaining to the prices of energy and security corresponding to the optimal schedules of the market-clearing process. The key result of this analysis establishes that involuntary load shedding is used after a contingency if and only if the expected marginal costs of scheduling reserves and deploying them are greater than the expected marginal costs of load shedding.

We then extend the model of electricity market-clearing with stochastic security by proposing a short-term electricity market-clearing formulation capable of accounting for non-dispatchable and intermittent power generation sources like wind power. We show how the electricity market-clearing model can take into account uncertainties in the next day/hours wind power generation predictions as well as those of the demand. Also, we demonstrate how the market-clearing formulation can integrate the scheduling of a large-scale centralized energy storage infrastructure.

Finally, we define rigorously the concept of the set of umbrella contingencies for security-constrained optimal power flow problems, a class of power system scheduling problems to which market-clearing with stochastic security belongs. We propose an identification

method to identify the members of this set by making use of the vector norms of the Lagrange multipliers associated with the post-contingency power balance relations. We suggest a heuristic contingency ranking rule based on those vector norms, and we argue that the proposed identification rule and ranking method can be of use to system operators when specifying reduced sets of contingencies for security-constrained market-clearing problems.

Résumé

Cette dissertation développe la formulation d'un problème d'ordonnancement à court terme d'un marché d'électricité soumis à des critères de sécurité stochastiques basés sur des mesures probabilistes comme, par exemple, la probabilité de délestage postcontingence ou la valeur probable de la grandeur de celui-ci. Ici, les mesures probabilistes sont calculées pour des ensembles de contingences présélectionnées pouvant inclure des défaillances aléatoires des groupes de production, des lignes de transport ainsi qu'à d'importantes déviations de la demande par rapport à sa prévision. Nous démontrons qu'en pénalisant économiquement la valeur probable du délestage postcontingence, il est possible d'imposer des critères de réserves opérationnelles basés sur le risque économique de délestage en lieu et place des critères de réserves empiriques utilisés par l'industrie. Le processus d'ordonnancement optimal du marché d'électricité gagne donc en flexibilité, car il est ainsi possible de balancer les valeurs probables des coûts correspondants: (i) aux actions préventives associées au mode d'opération en précontingence spécifiant les décisions relatives à la synchronisation et aux points de consigne des groupes de production, l'ordonnancement de la demande, ainsi qu'aux niveaux des réserves opérationnelles; (ii) aux corrections postcontingences pouvant inclure des changements d'état de synchronisation et de points de consigne des groupes de production; et (iii) au délestage postcontingence qu'il soit volontaire ou involontaire. Des études de cas démontrent que la méthode d'ordonnancement stochastique proposée permet de réduire de manière non négligeable les coûts sociaux reliés à la production et au transport de l'électricité, tout en permettant aux consommateurs de continuer de jouir d'un approvisionnement électrique fiable en fonction des coûts sociaux associés aux événements de délestage.

Nous démontrons ensuite une série de résultats théoriques reliés aux prix de l'énergie et de la sécurité associés à l'ordonnancement optimal d'un marché d'électricité soumis à un critère de sécurité stochastique. Un des résultats clés de cette analyse établit que des actions de délestage involontaires sont utilisées si et seulement si la valeur probable des coûts marginaux associés à l'ordonnancement et au déploiement des réserves opérationnelles est supérieure à celle des coûts marginaux de ces actions de délestage.

La formulation du problème d'ordonnancement stochastique à court terme est étendue afin d'accommoder le traitement de l'incertitude dans les prévisions de sources d'énergie électrique intermittentes, et pour lesquelles il est pratiquement impossible de spécifier un

point de consigne de production, comme l'énergie provenant d'éoliennes. Nous démontrons comment modéliser l'incertitude dans les prévisions de production éolienne et de celle de la demande afin de pouvoir adapter la formulation déjà développée à ce type d'incertitude. De plus, nous indiquons comment cette nouvelle mouture du problème d'ordonnancement stochastique peut facilement intégrer des systèmes de stockage d'énergie à grande échelle.

En dernier lieu, nous définissons de manière rigoureuse la notion de contingence parapluie dans le contexte d'un problème général d'écoulement de puissance optimal avec contraintes de sécurité, où, de fait, les problèmes d'ordonnancement définis dans cette dissertation forment une sous-classe. Nous proposons une règle servant à identifier ces contingences parapluie à partir des normes vectorielles des multiplicateurs de Lagrange associés aux contraintes nodales d'équilibre de puissance postcontingence. De plus, nous suggérons une règle de classement de ces contingences. Ces règles pourraient constituer les bases d'un outil utile aux opérateurs de réseaux afin de réduire le nombre de contingences devant être traitées au moment d'évaluer la sécurité d'un réseau.

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List of Acronyms

AGC	Automatic generation control
BTU	British thermal unit (1 BTU $\approx$ 1.06 kJ)
DA	Day-ahead
DOE	United States Department of Energy
ED	Economic dispatch
ELNS	Expected load not served
FERC	United States Federal Energy Regulatory Commission
HA	Hour-ahead
IEEE RTS	IEEE Reliability Test System
ISO	Independent system operator
KKT	Karush-Kuhn-Tucker
LOLE	Loss-of-load expectation
LOLP	Loss-of-load probability
LP	Linear programming
MILP	Mixed-integer linear programming
NERC	North American Electric Reliability Council
PJM	Pennsylvania-New Jersey-Maryland Interconnection
RV	Random variable
SCMC	Security-constrained market-clearing
SCOPF	Security-constrained optimal power flow
VOLL	Value of lost load
VSS	Value of the stochastic solution
WP	Wind power

List of Symbols

The main symbols used throughout this dissertation are defined below. Others will be defined as required in the text.

Indices

i	Index of generators running from 1 to I.
j	Index of discrete realizations of the net load forecast error running from 1 to J.
k	Index of contingencies or net load forecast error scenarios running from 1 to K.
ℓ	Index of transmission lines running from 1 to L.
m	Index of buses or demands running from 1 to M.
t	Index of time periods running from 1 to T.
τ	Index of contingency occurrence intervals running from 1 to T.

Sets

$\mathbb{B}$	Set of binary numbers.
$\mathbb{R}$	Set of real numbers.
$\mathcal{A}_m$	Set of generators located at bus m.
$\mathcal{B}_m$	Set of transmission lines connected to bus m.
$\mathcal{C}_k$	Set of failed components corresponding to contingency k.
$\mathcal{D}_{mt}$	Feasible operating region of demand at bus m during period t.
$\mathcal{G}_{it}$	Feasible operating region of generator i during period t.
$\mathcal{K}$	Set of contingencies.

$\mathcal{S}_k$	Set of ordered indices defining net load forecast error scenario k.
$\mathcal{T}$	Set collecting net load forecast error scenarios, *i.e.* net load forecast error scenario tree.
$\mathcal{U}$	Set of umbrella contingencies.

Variables

d_{mt}	Demand at bus m during period t.
e_t	Energy stored in an energy storage system during period t.
g_{it}	Power output of generator i during period t.
l_{mt}	Involuntarily shed load at bus m during period t.
n_t	Net load during period t.
r_{it}^{up}	Spinning reserve up provided by generator i during period t.
r_{it}^{dn}	Spinning reserve down provided by generator i during period t.
r_{mt}^{up}	Spinning reserve up provided by demand at bus m during period t.
r_{mt}^{dn}	Spinning reserve down provided by demand at bus m during period t.
$\tilde{r}_{it}^{up}$	Non-spinning reserve up provided by generator i during period t.
$\tilde{r}_{it}^{dn}$	Non-spinning reserve down provided by generator i during period t.
s_t	Wind power generation spillage during period t.
u_{it}	Binary variable (equals 1 if generator i is online during period t; 0 otherwise).
w_t	Wind power generation during period t.
y_t	Binary variable (equals 1 when an energy storage system is being charged during period t; 0 otherwise).
z_t^c	Rate of charge of an energy storage system during period t.
z_t^d	Rate of discharge of an energy storage system during period t.
δ_{mt}	Voltage angle of bus m during period t.
ψ	Binary variable (equals 1 if a generation outage combination leads to loss-of-load; 0 otherwise).

Random Variables

ξ_i	Bernoulli random variable indicating the availability of generator i (equals 1 if generator i is available; 0 otherwise).

ϕ_i	Bernoulli random variable indicating whether generator i is simultaneously online and available (equals 1 if generator i is online and available; 0 otherwise).
θ_{mt}	Random variable modeling the load forecast error of demand m during period t.
θ_{nt}	Random variable modeling the net load forecast error during period t.
θ_{wt}	Random variable modeling the wind power generation forecast error during period t.

Functions

$B_d(\cdot)$	Demand-side energy consumption benefit function.
$C_g(\cdot)$	Generation-side energy production cost function.
$C_r(\cdot)$	Reserve supply cost function.
$C_s(\cdot)$	Energy storage operating cost function.
$f_\ell(\cdot)$	Function expressing the power flow on transmission line ℓ.
$F(\cdot)$	Cumulative probability distribution function.
$G(\cdot)$	Function describing some general feasible set.
$H(\cdot)$	Function describing the nodal power balance.
$W(\cdot)$	Social cost function.

Parameters

$e^{\max}$	Maximum energy storage capacity.
$f_\ell^{\max}$	Maximum power flow allowable on transmission line ℓ.
$g_i^{\max}$	Maximum generation output of generator i.
$g_i^{\min}$	Minimum generation output of generator i.
$p(0)$	Probability that no contingencies occur.
$p(k,\tau)$	Probability that no contingencies occur during the scheduling horizon except for contingency k which occurs during interval τ.
$p_t(k)$	Probability of net load forecast error scenario k during period t.
v_{mt}	Value of lost load at bus m during period t.
$z^{\max}$	Maximum energy storage charging/discharging rate.
$\bar{z}^{\max}$	Maximum energy storage ramping rate.

γ	Value of energy stored at the end of a scheduling horizon.
ε	Umbrella contingency determination cutoff threshold.
η^c	Charging efficiency of an energy storage system.
η^d	Discharging efficiency of an energy storage system.
σ_{mt}	Standard deviation of the load forecast error associated with demand m during period t.
σ_{nt}	Standard deviation of the net load forecast error during period t.
σ_{wt}	Standard deviation of the wind power generation forecast error during period t.
Δ	Time span between two time periods t and $t+1$.

Lagrange Multipliers and Prices

μ_{mt}	Lagrange multiplier associated with the power balance constraint at bus m during period t.
σ	Lagrange multiplier associated with an inequality constraint.
λ^E_{mt}	Marginal price of energy at bus m during period t.
λ^S_{mt}	Marginal price of security at bus m during period t.

Remarks

- A variable, function or parameter written in bold without one or more indices is a vector form representing the corresponding quantity. For example, the symbol $\boldsymbol{\delta}_t$ represents the vector of bus voltage angles during period t. In the same fashion, the symbol $\boldsymbol{\delta}$ represents the vector of bus voltage angles over the entire duration of the planning horizon.
- In Chapter 2, when augmented with the argument (k, τ), the above parameters, variables and functions represent their value given that contingency k has occurred within interval τ. A similar remark applies to the argument (k) whereby the time of failure aspect is neglected, or embedded within the contingency index as in Chapters 3 and 5. Symbols written without any extra argument denote pre-contingency values, that is $k = 0$.
- In Chapter 4, when augmented with the argument (k), the above parameters, vari-

ables and functions represent their value given that the net load error random variable follows scenario $\mathcal{S}_k$.

- A parameter or a variable written with a "hat" (*e.g.* $\hat{x}$) designates the forecasted value of the parameter or the expected value of that variable.

Il n'y a pas de honte à préférer le bonheur.

Albert Camus, 1913–1960

Chapter 1

Introduction

They also serve who only stand and wait.

On His Blindness
John Milton, 1608–1674

1.1 Background

1.1.1 Schweppe's vision

In the aftermath of the 1977 blackout that had left New York City in the dark for almost a day, Fred C. Schweppe published his famous *IEEE Spectrum* article: "Power Systems '2000': Hierarchical Control Strategies" [1]. In this paper, and in a follow-up article published in 1982 with his MIT colleagues Tabors and Kirtley [2], he envisioned the evolution of the electric power system operational paradigm from the centralized "command-and-control" approach, that had driven the industry since its beginnings, towards one which would be more decentralized and where the lightly-coordinated self-regulating actions of local devices would be sufficient to keep the overall system in a secure and balanced state.

A concrete consequence of this proposed paradigm was that the then current business model of the electric utility—grouping together the generation, transmission and distribution business functions—would lose its appeal. As consumers and generators would be able to self-coordinate through some common communication channels without significant needs for centralized controls, the tightly-regulated vertically-integrated utility structure would no longer be required. In fact, power systems in the year 2000 would self-regulate through the straightforward price signals broadcasted minute-by-minute over those common communication channels so to precisely mediate the balance of supply and demand for electricity. In other words, an electricity market would do the job the utilities had done for nearly a century. The concept of *spot pricing of electricity* [3] was born.

During the 1980's, the world political scene, in the United States of America and the United Kingdom most notably, fostered the restructuring of traditionally state-regulated industries, airlines and telecommunications being the best examples, through the introduction of competition. The electric industry was not neglected in this restructuring wave as some of the principles behind spot pricing of electricity were introduced in the early electricity pool markets formed in Chile (1982), England and Wales (1990) and Norway (1991–1992) [4]. Likewise, technological innovations in the fields of microelectronics, computing, telecommunications and electricity generation—especially with the appearance of the highly flexible and efficient commercial-grade combustion turbines combined with the soaring fuel prices in the late 1970's and early 1980's—gave further credence to the introduction of competition in the electricity industry.

Restructuring in the United States of America

In 1992, with the passing into law of the Energy Policy Act, the United States of America embarked the electricity restructuring train. The issuing in April 1996 of Orders 888 and 889 [5,6] by the Federal Energy Regulatory Commission (FERC) officialized the move that would shake up the North American electric industry like it had never been before.

The legislators of the energy-hungry state of California were among the first to see in the introduction of competition a possible way to improve the economic efficiency of its electricity sector [7]. The stakeholders there agreed on a somewhat *pure* market model [4] where a variety of generation companies, marketers, load serving entities and power exchanges would interact in a highly decentralized way with the least centralized intervention of the grid operator—the California Independent System Operator (ISO). Under this operational paradigm, the generation and consumption schedules are obtained through prior contract negotiations or through energy-only market-clearing processes run by separate power exchanges. It is only after the energy schedules have been set that the ISO has to ensure, with as little schedule manipulations as possible, the integrity of grid operation. However, many flawed market rules combined with a variety of factors [8] led to (i) several episodes of state-wide rolling blackouts; (ii) many days during which the grid was operated below the prescribed reliability standards; and, (iii) the bankruptcy of the largest investor-owned utility in the United States (Pacific Gas and Electric Company).

In other parts of the United States, the transition operated a bit more smoothly especially in areas where there were already existing regional electricity pooling structures prior to restructuring [for example, in New York, New England and the Pennsylvania-New Jersey-Maryland Interconnection (PJM)]. The operational models followed in these markets were much closer to the "command-and-control" approach associated with the old regime. These regional markets were set up under what is known as the "pool" model [4, 9–11] wherein generation and demand are centrally scheduled by the ISO based on economic offers for the production of energy and bids for the consumption of that energy. Moreover, under these market structures, the ISO schedules the electricity market with the objective of meeting a number of system security requirements like transmission line flow limits, bus voltage limits and so on [9]. The ISO also has responsibilities in balancing supply and demand in real time and has to calculate locational (nodal) marginal prices to be used later for financial settlement purposes. This operational model came to be the template for what became known as Standard Market Design [12, 13], an initiative launched in 2002 by FERC in response to the California debacle. In July of 2005, this initiative was terminated as a result of voluntary measures fostering improved reliability adopted across the industry.

Restructuring in Canada

In Canada, Alberta (in 2000) and Ontario (in 2002) are the only provinces that have truly introduced competition in the wholesale electricity sector following the divesture of their state-owned generation capacity. In Alberta, the industry reforms led to an improvement of the province's generation mix through a number of capacity additions mostly as gas-fired cogeneration facilities. In Ontario, however, the opening of the market came at a time of tight capacity margins, which drove prices charged to end-consumers to unprecedented levels. Public pressure eventually forced the Conservative government in Queen's Park to adopt the Electricity Supply, Pricing and Conservation Act in November 2002. The Act holds a series of initiatives geared to protect residential, small business consumers as well as farmers from the high prices of electricity.

Nonetheless, in the provinces massively exporting electricity to the United States—

Québec, Manitoba and British Columbia, for the most part—, some level of restructuring of the local state-owned utilities was necessary in the wake of restructuring south of the border. To be able to keep on selling in the American markets, the Canadian utilities had to reciprocally open their grids and allow trading activities between any qualifying generating and load-serving entity irrespective of their location. For example, in the case of Hydro-Québec, this led to the breakup in 1997 of its vertically-integrated corporate structure into three separate and functionally independent entities: Hydro-Québec Production (generation), Hydro-Québec TransÉnergie (transmission) and Hydro-Québec Distribution (distribution and retail).

The competitive utopia

With hindsight, the electricity industry has changed significantly since Schweppe's first paper. However, it has not evolved truly in the direction envisioned. The current operational paradigm of electricity markets is still far from the decentralized operational ideal put forward nearly 30 years ago. Indeed, in most markets the process that was once termed "deregulation" has often turned into an explosion in the amount of new "regulation" so desperately needed to keep the lights on as well as those computers running our digital society.

The reasons why Schweppe's idea is still a utopia today are surely many. In our opinion, however, the root cause lies essentially in the impossibility for consumers to respond in a timely fashion to the changing conditions of the market considering that: (i) it is necessary to minutely balance supply and demand second-by-second since electricity is generally not storable; (ii) there cannot be real-time response of consumers to electricity prices since there are still no truly effective common communication channels linking generators and loads; and to exacerbate these problems even more, (iii) electrical energy is poorly substitutable *vis-à-vis* other sources of energy. As Joskow and Tirole [14] put it, in such a market, controls based on prices only are not fast enough nor sufficient; therefore, it remains that demand-side rationing, in the form of load shedding (*i.e.* blackouts), are inevitable.

1.1.2 From blackout to blackout

Since grids opened up to accommodate competition, the volume of electricity trades has increased significantly at the same time as trading structures became ever more complex, especially with the increasing distances associated with power injection and delivery points. For instance, there were increases in power flows scheduled on inter-control area tie lines which were originally designed to serve system reliability purposes as emergency power makeshifts. Likewise, as electricity flows according to Kirchoff's Laws and Ohm's Law, the undesirable phenomenon of loop flows from neighboring systems has rendered the reliability enforcement tasks of ISOs even more complicated.

Notwithstanding the steady increase in the complexity of the reliability enforcement tasks faced by the ISOs, over the last 10 years very little research and development efforts were devoted to power system reliability in a market environment. In fact, it took yet another major blackout, that of August 14th, 2003, which left in the dark millions of consumers in Ontario, New York, the Midwest and parts of New England [15], to persuade the industry, governments and the academic community to address power system reliability in market contexts. This wake up call repeated itself shortly after with blackouts that affected major areas in several European countries [16–19].

This research work constitutes a direct response to those costly failures as we examine and propose novel and improved tools for the secure scheduling of power systems in a market context. It is worthy to note that among the 46 recommendations of the joint Task Force set up by the United States Department of Energy (DOE) and Natural Resources Canada to investigate the August 14th, 2003 blackout [15], the research work reported in this dissertation directly addresses:

Recommendation 9: Integrate a "reliability impact" consideration into the regulatory decision-making process.

Recommendation 13: DOE should expand its research programs on reliability-related tools and technologies.

Recommendation 22: Evaluate and adopt better real-time tools for operators and reliability coordinators.

Recommendation 30: Clarify criteria for identification of operationally critical facilities, and improve dissemination of updated information on unplanned outages.

1.1.3 Power system operations planning timeline

It is instructive to examine the chronology of steps involved with power system operations planning as illustrated in Fig. 1.1.

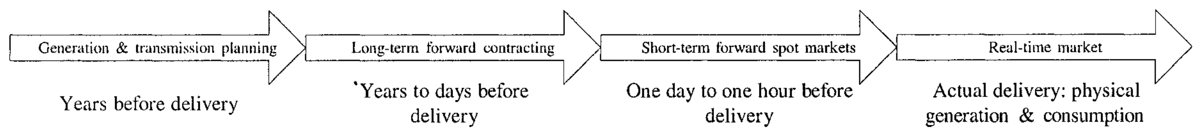

Fig. 1.1 Power system operations planning timeline

Generation and transmission capacity planning

The generation and transmission capacity planning stage does not truly belong in the operations planning chronology. However, it represents a critical stage that cannot be ignored because it greatly influences the viability of a power system in the future. It is well understood that under-investment in generation and transmission capacity can have dramatic impacts on society through higher prices and reduced reliability. The opposite situation, whereby there is over-investment in new capacity, may be damaging to society too as it may prove to be more difficult for generation and transmission owners to recover their investments. In fact, the central issue of power system planning—both in competitive and regulated settings—will always remain the difficulty in trying to smooth out those "boom and bust" investment cycles [10].

Long-term forward contracting

With the introduction of competition in the electricity business, wholly new opportunities for electricity trading have arisen. Standardized forward and futures contracts for electricity are now widely available,[1] while a variety of electricity-based derivative products [20] are now available through specialized brokers and marketers. Likewise, traditional bilateral contracts negotiated privately between consumers and generators are commonplace.

These contractual arrangements constitute a critical part of any good risk management practice for a generator or a load-serving entity. For generators, forward contracts provide

[1] For example, a forward electricity contract specifies a quantity of energy to be delivered at a given location and time for some price.

sure future cash inflows. On the other hand, with theses contracts consumers can lock in a good part of their predicted load at a fixed price. The more exotic instruments like options offer other useful hedging properties, but at a generally higher transaction cost.

Short-term forward market-clearing

As the time to delivery approaches (24 to one hour before delivery), consumers can better predict how much energy they will likely consume on top of or under their previous forward contracting agreements. This is why in many systems short-term forward electricity markets have been set up [4, 10]. These markets are centrally-run by the system operator that schedules the generators and consumers based on their respective offers and bids through an *electricity market-clearing* procedure [11]. Next, we discuss the two flavors under which these markets are found: *day-ahead* (DA) and *hour-ahead* (HA) markets [10, 11].

Day-ahead market-clearing

Fig. 1.2 shows the general DA market chronology. First, given the information previously broadcasted by the system operator about the predicted power system conditions, the generators and the consumers prepare and submit their offerings and bids for the next day. Depending on the degree of modeling involved in the market-clearing procedure, the complexity of those offerings and bids will vary. Generation offerings can include incremental energy cost curves, startup costs, ramping and generation capacity limits, reserve offering rates and limits and so on. Demand-side bids can be made up of incremental energy benefit curves and elasticity limits. Likewise, when possible, voluntary reserve offerings can be made by the consumers [21].

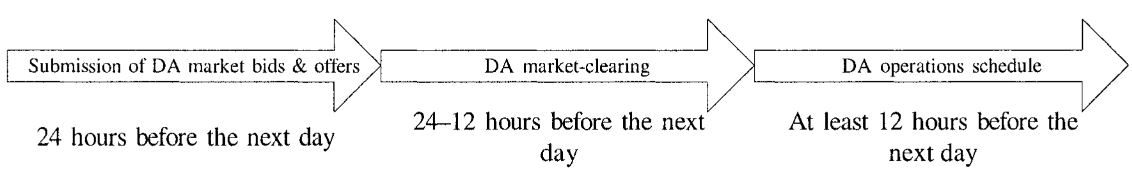

Fig. 1.2 Day-ahead market timeline

Next up, the system operator runs the market-clearing procedure. In North America and in other parts of the world (notably New Zealand and Spain), DA electricity market-clearing is based on a security-constrained unit commitment [22–31]. Specifically, security-constrained unit commitment-based DA markets determine for the next day the hourly:

- Generator on/off status and power output levels;
- Demand consumption levels; and,
- Generator- and demand-side reserve supplies.

The current DA market-clearing procedures are run with the objective of maximizing the net social welfare (or equally minimizing the net social cost) associated with the generation and the consumption of energy diminished by the costs of scheduling reserves needed to respond to unpredictable contingencies like generation failures. The market-clearing problems have as their main constraint the requirement that the supply and the demand match at each node (bus) of the grid for each of the 24 hours of the next day, while all transmission line flows are within acceptable limits. The other constraints of DA electricity market-clearing problems generally include the technological limits of the generators and the consumers.

Market-clearing problems are therefore large-scale mathematical programming problems solved on high-end computation servers running custom-purpose software tools. The speed of computation is generally an issue as there are strict time limits under which the system operator must clear the market. Likewise, the relative optimality of the schedules obtained is critical because very important sums of money are at stake. In addition to the generation and demand schedules, the DA market-clearing produces sets of bus marginal prices [10] that indicate the marginal value of electricity at each bus for each hour of the next day.

Finally, once the market schedules and prices have been obtained, they are sent back to the respective generators and demands.

Hour-ahead market-clearing

Getting even closer to the actual electricity delivery time, in some power systems it may be possible to adjust schedules through an hour-ahead market. The HA markets generally work on the same principles as the DA markets but generally without unit commitment. As Fig. 1.3 illustrates, the sequence applying to DA markets is still valid—that is offering and bidding, market-clearing then followed by the final operation and pricing schedules transmission to generators and demands.

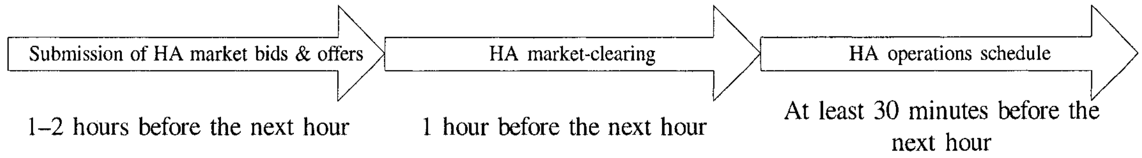

Fig. 1.3 Hour-ahead market timeline

Real-time market

As its name implies, the real-time market carries the function of balancing electricity supply and demand minute-by-minute. The basic assumption here is that the real-time market is a "deviations" market that makes up for the differences between the actual generation and consumption and the set points scheduled in the previously-cleared DA and HA markets.

Economic dispatch and secondary regulation

Economic dispatch (ED) [11, 32] forms the core of the real-time market. At regular time intervals (every five minutes in PJM, for instance), an ED is run to update the set points of the generators and those of the loads that are dispatchable. The results of the ED are based on the economic merit of adjustment offers and bids reflecting the costs and/or benefits associated with the deviations. Like the DA and HA markets, the real-time market derives bus marginal prices of electricity, which are used to remunerate the generators and charge the demands.

In between ED runs, automatic generation control (AGC) [32–34], also known under the name of secondary regulation, keeps the power system balanced as it automatically orders a subset of the synchronized generators to modify their generation set points proportionally to some participation factors. These control orders follow the slowly-varying changes in the demand with the goal of returning the area control error to zero [32, 33, 35]. The control actions associated with AGC typically involve time constants of the order of several minutes.

Operation under contingencies

However, during real-time operation, the controls based on ED and AGC are usually overridden in the event that a contingency occurs. In this dissertation, a *contingency* is defined as:

Definition 1.1 (Contingency). A *contingency* is an unexpected outage of a one or more pieces of equipment, such as generators and lines, as well as large, unexpected load variations.

In the aftermath of contingencies, other load-generation control mechanisms, primary and tertiary regulation, are called in through automatic or manual *deployment* of *reserve services*.

Definition 1.2 (Reserve service). A *reserve service* or simply *reserve* is the capability above or below the pre-contingency production or consumption set point available to respond voluntarily to contingencies within a given time frame.

Definition 1.3 (Reserve deployment). *Reserve deployment* is the set of post-contingency actions through which reserve services are converted into actual energy production or consumption.

Next, we describe briefly the control objectives that correspond to the primary and tertiary regulation intervals and their associated reserve deployment actions.

Immediate response: primary regulation

At the initiating instant of a contingency, power imbalances are created across the grid. If persistent, such power imbalances eventually lead to deviations in the frequency of the spinning generators. During the primary regulation interval, spinning generators adjust their power output automatically in response to deviations from the nominal system frequency [32, 36]. Such reserve deployment actions are fast and serve to keep the system frequency from going adrift within 5 to 10 seconds of a contingency. In cases when this fails to happen or the frequency excursion goes beyond the narrow band centered about the nominal frequency, automatic load shedding and/or generation rejection may follow.

One should note that loads too respond to changes in frequency; therefore, the consumers end up participating during the primary regulation interval. However, they usually do so without knowing and unintentionally [32]. In most systems, this demand-side contribution is entirely ignored.

Fig. 1.4 shows a typical contingency recovery process happening in real time. In this example, an ED is run every 10 minutes while AGC follows the load variations in between the ED runs. At some time between $t = 0$ and $t = 10$ minutes a contingency occurs (marked by the X on the timeline in Fig. 1.4). In the initiating instants of the contingency, primary regulation actions are deployed to keep the frequency of the system from drifting outside its allowed range.

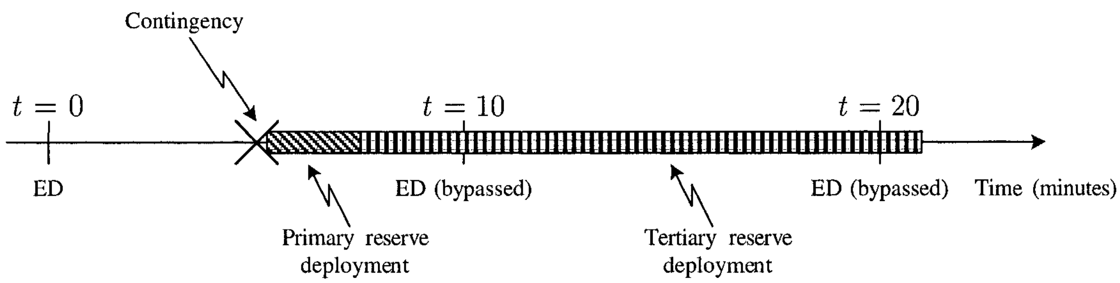

Fig. 1.4 Contingency recovery process in the real-time market

Steady-state recovery: tertiary regulation

Within 10 to 20 minutes after the initiating moment of a contingency, at the end of the tertiary regulation interval, generation and demand levels have been adjusted through appropriate reserve deployment actions to ensure that (i) power balances at every network bus; (ii) all transmission line flows are within their prescribed limits;[2] (iii) the area control error has been nullified; and, (iv) all generators and demands operate within their feasible technical limits. In other words, the reserve deployment actions steer the power system back into a "steady-state" mode of operation wherein all steady-state operational constraints are met. We point out that the tertiary regulation interval, because of its longer range of action, allows for a broader involvement of demand-side and non-spinning generation-side reserve resources.

Coming back to the example in Fig. 1.4, we see that the tertiary regulation actions start their deployment after the primary regulation actions stabilized the system frequency. Thus, during the next 15 minutes, the generators and demands providing tertiary reserve services modify their set points as instructed by the system operator, while those not providing reserve services are asked to return to their pre-contingency schedules. During

[2] We note here that restrictions on transmission line flows are not strictly enforced during the primary regulation interval unlike during the tertiary interval. This is so for two reasons: (i) lines can be slightly overloaded for short periods without significant risk; and, (ii) the controls associated with the primary regulation interval cannot enforce the line limits because they do not have sufficient degrees of freedom.

that interval, the ED and the secondary regulation functions are bypassed until the end of the deployment of the appropriate reserves.

One should note that, in the aftermath of a contingency that leads to the deployment of some reserve services, the system operator has to evaluate whether it is necessary to schedule more reserves services and/or to distribute them differently among the generators and demands. Of course, this evaluation is conducted with the objective of ensuring the security of the system if another contingency were to happen at a later time.

As a final note of caution, we point out that in this dissertation, we do not adhere strictly to the definitions of reserves put forward by the North American Electric Reliability Council (NERC) [35], which classifies reserves as either *spinning* or *supplemental*. In NERC's definition, spinning reserves are provided by online generators already synchronized to the grid. They (i) respond automatically to major contingencies to keep the system in balance—that is, they provide primary regulation—; and, (ii) they react to the normal load variations through AGC—that is, they provide secondary regulation. On the other hand, the supply of supplemental reserves is not restricted to synchronized (spinning) generators only. Supplemental reserves are used to respond to contingencies over longer time horizons of tens of minutes—that is, they provide tertiary regulation.

Here, in our definition of reserve (recall Definition 1.2), we do not pre-suppose any synchronization state for the generators and loads that provide the services. In fact, we adopt the philosophy laid down in [34] under which the reserves are classified by their response time delay rather than by the synchronization status of those providing them.[3]

1.2 Problem Identification

Hobbs *et al.* in [37] report that, according to a panel of leading industry and academic experts, fundamental research is required towards the treatment and the inclusion of uncertainty factors within short-term market-based power system scheduling problems (*i.e.* DA and HA markets). Also, according to the same panel, issues pertaining to the relia-

[3]From a practical point of view, however, both the NERC definitions and those of [34] could be considered to be equivalent since it is practically impossible for a non-spinning generator to respond to disturbances within the time frames of the primary and secondary regulation intervals.

bility[4] enforcement tasks of system operators constitute important aspects of short-term scheduling that have been neglected thus far.

Furthermore, as we just saw in Section 1.1, in the aftermath of the August 14th, 2003 blackout, the worldwide power systems operations planning community truly came to realize that system operators lacked the proper tools to address the risks associated with uncertainty and their impacts on system security and economics. The research work reported in this dissertation constitutes a step forward in the development of such tools. The gist of the work here is an investigation of the pertinence of making use of stochastic measures in formulating security criteria for short-term electricity market-clearing problems. A number of other related subproblems are also investigated; we introduce each of them in the following subsections.

1.2.1 Short-term security-constrained electricity market-clearing

Reserve-constrained electricity market-clearing

The requirements for tertiary reserve services[5] in short-term reserve-constrained electricity markets are generally set using deterministic rule-of-thumb type criteria. For instance, it is commonplace in the industry to require that enough reserve is available so to cover the loss of the largest generator—best known as the $N-1$ criterion—as in case of the Southern Zone of PJM [39]. Another common reserve criterion demands that the system operator schedules an amount of reserves greater than or equal to a fraction of the daily or hourly demand; in PJM again, the reserve requirement of its Western Zone is determined that way [39]. Others, like the Spanish system [40], use a hybrid approach whereby the required amount of tertiary reserve must be equal to the largest online generator plus two percent of the hourly predicted demand. We note, however, that such preventive security measures [41] do not take into consideration the probability of occurrence of the contingencies they need to

[4]In the power systems context, reliability is an umbrella term comprising two basic functional aspects: *adequacy* and *security* [38]. Adequacy refers to the ability of the power system to properly serve its load through enough supplies of generation and transmission. Security, on the other hand, which could be also called "short-term reliability" [38], is the ability of the power system to withstand disturbances in the course of its operation. As this dissertation is concerned essentially with short-term power system operational issues, we use the word *security* when referring to the notion of reliability.

[5]Everywhere in this dissertation, unless it is stated otherwise, when using the word "reserve" or the expression "reserve service" we refer to reserves that can be deployed during the tertiary regulation interval following a contingency.

cover. If these probabilities are sufficiently low, on the average, over-scheduling of reserve may result. On the other hand, if the probabilities of occurrence are high, the reserve requirements may not be sufficient.

Towards stochastic reserve criteria

Kirschen in [42] advocates that power system security assessment methods should be capable of appraising the "credibility" of outages as well as their "expected" effects through probabilistic means. It is generally well understood, however, that such stochastic security analysis methods are computationally costly because they require the evaluation of the probabilities and of the consequences of a large number of possible failure events. For reserve-constrained unit commitment and short-term electricity market-clearing problems, this combinatorial aspect represents the main restricting factor to their widespread use; this is well recognized in many contributions found in the literature [43–50]. In addition to the complicating computational aspect of probabilistic methods, often there are significant uncertainties associated with the statistical failure data available necessary to compute outage probabilities.

It is important to note that the idea behind using stochastic reserve criteria for short-term operations planning purposes is not entirely new. For instance, back in 1963 the authors of [43] defined the notion of *unit commitment risk* which was later formalized by Billinton and Allan [46]. This risk is a measure representing the probability of not meeting the predicted load. In this so-called *PJM method*, generating units are committed sequentially according to a pre-established priority list until the unit commitment risk is below some threshold. Guy in 1971 [44] went on to formulate a reserve-constrained unit commitment problem using the concept of a *security function*. Like the unit commitment risk, the security function evaluates the probability that a power system may be incapable of meeting its forecasted load. There, the reserve criterion is imposed by bounding from above the magnitude of the security function as part of an optimization problem. Dillon *et al.* [45] were the first to truly formulate a unit commitment problem with all its intricacies and a stochastic reserve criterion. However, because of its complexity, a significant number of implementation problems were left pending. Gooi *et al.* [47] went on to propose an iterative approach based on Lagrangian relaxation, while Flynn *et al.* [48] proposed a solution method that made use of neural networks and an augmented Lagrangian solution technique.

Chattopadhyay and Baldick in [49] correctly pointed out that all the methods proposed so far cannot represent efficiently the probability distribution of the discrete capacity outages, the capacity outage probability table [46], directly in terms of the unit commitment variables. These authors overcame this difficulty by proposing to approximate the discrete outage probability distribution by a continuous function of the unit commitment variables.

Deterministic/probabilistic approach

In light of the difficulties brought about by the computational burden and the uncertainties in the failure data, some authors have proposed to use *hybrid* deterministic/probabilistic security analysis methods rather than purely stochastic ones that require the full enumeration of the possible system states [50–52]. In those hybrid methods, rather than evaluating the probabilities and the expected outcomes of all possible failure modes, only limited sets of *a priori*-defined outage events are used to estimate those probabilities and expected outcomes.

For a civil engineering structural design problem, Castillo *et al.* [51] used a hybrid method to deal with the uncertainty in the failure probabilities (due to the lack of historical failure data) as well as to reduce the complex computational facets of stochastic structural failure metrics. Billinton and Mo in [52] proposed to use a deterministic/probabilistic approach for power system planning purposes. In their work, the authors attempted to evaluate the system-wide and local effects of a pre-specified set of failures affecting major power system components.

A deterministic/probabilistic approach for network-free single-period reserve-constrained unit commitment is proposed by Bouffard and Galiana in [50] (and is further extended with Conejo in [53,54]). In [50], a stochastic reserve criterion is imposed by bounding from above a deterministic/probabilistic security metric. As this proposal forms the preliminary basis of the investigation reported in this dissertation and in [53, 54], we examine it in details next.

Security metrics as explicit functions of unit commitment variables

The proposed approach in [50] assumes that the $i = 1, \dots, I$ generating units in the power system can be scheduled on or off by selecting the value of the variables $u_i \in \mathbb{B} = \{0, 1\}$

$$u_i = \begin{cases} 1 & \text{if on,} \\ 0 & \text{otherwise.} \end{cases} \tag{1.1}$$

Independently of their scheduling status, the availability of the $i = 1, \dots, I$ generators varies randomly and may be modeled using Bernoulli random variables (RV) $\xi_i \in \mathbb{B}$

$$\xi_i = \begin{cases} 1 & \text{if available,} \\ 0 & \text{otherwise,} \end{cases} \tag{1.2}$$

where, as discussed in Appendix A.1, the probabilities associated with the two possible states[6] of the RVs ξ_i can be computed from historical data; see, for instance, the generation and line outage data collected by the Canadian Electricity Association since 1977 [55, 56].

Given the commitment status of generators and their random availability, the probabilities of new RVs $\phi_i = u_i \xi_i$, for $i = 1, \dots, I$, satisfy the conditions

$$P\left[\phi_i \geq u_i\right] = 1 - u_i U_i, \tag{1.3}$$

and

$$P\left[\phi_i < u_i\right] = u_i U_i, \tag{1.4}$$

where U_i is the forced-outage rate [46] of generator i. In plain words, (1.3) indicates that the probability that a generator i is available when scheduled on equals $1 - U_i$, while (1.4) shows that the probability that generator i is unavailable when it is scheduled on is U_i. It is clear here that these relationships provide an explicit way to model the outage probabilities of the generators in terms of the unit commitment variables, u_i. With these individual generator probabilities, it is possible to calculate the probability of any combination of generator

[6] Availability models based on multiple derated states are also possible [46]. However, for simplicity here we use a two-state model only.

outages $\mathcal{C} \subseteq \{1, \ldots, I\}$—assuming independent failures—from the expression

$$p(\mathcal{C}) = P\left[\{\phi_i \geq u_i;\ i \notin \mathcal{C}\} \cap \{\phi_i < u_i;\ i \in \mathcal{C}\}\right] = \prod_{i \in \mathcal{C}} u_i U_i \prod_{i \notin \mathcal{C}} (1 - u_i U_i). \tag{1.5}$$

We note that the above expression is nonlinear as it can be expanded into a polynomial of degree I in the unit commitment variables.

The *loss-of-load probability* (LOLP) measures the extent up to which a system generation schedule may not be able to meet the scheduled demand, that is

$$LOLP = P\left[\sum_{i=1}^{I} \phi_i (g_i + r_i) < d\right], \tag{1.6}$$

where the variables g_i and r_i represent respectively the generation output and the reserve level of generator i, while d is the scheduled level of demand. So far, we have shown how to get the probabilities of the random variables ϕ_i; however, it remains to be seen how we can determine effectively which of the generator outages combinations $\mathcal{C}$ actually lead to loss-of-load. In [50], it is proposed to indicate whether a given combination of generator outages $\mathcal{C}$ leads to loss-of-load using variables $\psi(\mathcal{C}) \in \mathbb{B}$. These are forced to satisfy

$$\frac{d - \sum_{i \notin \mathcal{C}} (g_i + r_i)}{\sum_{i=1}^{I} g_i^{\max}} \leq \psi(\mathcal{C}) \leq 1 + \frac{d - \sum_{i \notin \mathcal{C}} (g_i + r_i)}{\sum_{i=1}^{I} g_i^{\max}}, \tag{1.7}$$

where the parameters $g_i^{\max}$ are the upper generation limits of the generators $i = 1, \ldots, I$.

By inspection of (1.7), the variable $\psi(\mathcal{C})$ takes the value 1 if the simultaneous unavailability of the generators $i \in \mathcal{C}$ leads to loss-of-load, while it takes the value 0 otherwise. Assuming that $\sum_{i=1}^{I} g_i^{\max} > d$, whenever the load is over the available scheduled generation and reserve, that is $d - \sum_{i \notin \mathcal{C}} (g_i + r_i) > 0$, then the left-hand side of (1.7) is strictly positive while its right-hand side is over 1. Therefore, the binary nature of the variable $\psi(\mathcal{C})$ forces it to assume the value 1. A similar argument can be followed for the opposite case for which $\psi(\mathcal{C}) = 0$.

Thus, given that now we have explicit expressions for the all possible outage probabilities $p(\mathcal{C})$ and loss-of-load indicator variables $\psi(\mathcal{C})$, the LOLP can be written explicitly as the summation, over all possible subsets of generator outages $\mathcal{C} \subseteq \{1,\ldots,I\}$, of the products $p(\mathcal{C})\psi(\mathcal{C})$

$$LOLP = \sum_{\mathcal{C}\subseteq\{1,\ldots,I\}} p(\mathcal{C})\psi(\mathcal{C}). \tag{1.8}$$

We notice that (i) the LOLP is a nonlinear function of the unit commitment variables, through the probabilities $p(\mathcal{C})$ and of the loss-of-load indicator variables $\psi(\mathcal{C})$; and, (ii) the computation of the LOLP involves the calculation of all probabilities and loss-of-load indicator variables for all possible outage combinations.

An alternative to the LOLP security metric is the *expected load not served* (ELNS), which we express mathematically as

$$ELNS = \sum_{\mathcal{C}\subseteq\{1,\ldots,I\}} p(\mathcal{C})\psi(\mathcal{C})l(\mathcal{C}). \tag{1.9}$$

In (1.9), the variable $l(\mathcal{C}) \in \mathbb{R}$ measures the amount of load shed (when positive) or the remaining reserve capacity (when negative) under some outage combination $\mathcal{C}$

$$l(\mathcal{C}) = d - \sum_{i\notin\mathcal{C}}(g_i + r_i). \tag{1.10}$$

We can rewrite (1.10) knowing that $\sum_{i\notin\mathcal{C}}(g_i + r_i) = \sum_{i=1}^{I} g_i + \sum_{i=1}^{I} r_i - \sum_{i\in\mathcal{C}}(g_i + r_i)$

$$l(\mathcal{C}) = d - \sum_{i=1}^{I} g_i - \sum_{i=1}^{I} r_i + \sum_{i\in\mathcal{C}}(g_i + r_i). \tag{1.11}$$

Now, using the fact that $d - \sum_{i=1}^{I} g_i = 0$ under pre-contingency conditions, we can therefore reexpress $l(\mathcal{C})$ more simply as

$$l(\mathcal{C}) = \sum_{i\in\mathcal{C}}(g_i + r_i) - \sum_{i=1}^{I} r_i. \tag{1.12}$$

As its name implies, the ELNS measures the average load lost during load shedding events. For instance, under the outage of the generators $i \in \mathcal{C}$, if load is shed we have $\psi(\mathcal{C}) = 1$ and the quantity $l(\mathcal{C}) > 0$ so that the expected load lost for that combination is $p(\mathcal{C})\psi(\mathcal{C})l(\mathcal{C}) = p(\mathcal{C})l(\mathcal{C}) > 0$; otherwise, we have $\psi(\mathcal{C}) = 0$ and $l(\mathcal{C}) \leq 0$ such that the product $p(\mathcal{C})\psi(\mathcal{C})l(\mathcal{C}) = 0$. Like in the case of the LOLP, the computation of the ELNS requires the calculation of a large number of variables and of their products.

As mentioned before, in [50] it is proposed to impose a stochastic reserve criterion by bounding the LOLP or the ELNS security metrics from above. However, it is recognized that in their current forms, these two metrics are not computationally suitable for current mixed-integer linear solvers because they involve the calculation of products of variables and, moreover, because they require the computation of all outage combinations, whose number grows exponentially with the size of the generating system.

This is where the notion of *hybrid* probabilistic/deterministic security metrics is introduced in this model. In [50] the hybrid security criteria are determined by bounding from above the probability of losing load (or the expected load not served) caused by single and double generation outages only in a way analogous to the deterministic $N-1$ and $N-2$ criteria. Nevertheless, since these reduced-order approximate security metrics still incorporate a measure of the probability of the outage events, they retain part of the desirable probabilistic properties of the *true* LOLP and ELNS.

Other means are proposed in [50] to reduce the computational complexity of the stochastic metrics. First, it is proposed to retain only the lowest-order terms when computing the outage probabilities $p(\mathcal{C})$; this further approximation leads to an over-estimation of the outage probabilities

$$p(\mathcal{C}) \leq \prod_{i \in \mathcal{C}} u_i U_i. \tag{1.13}$$

In addition, to eliminate the products of binary variables and of mixed binary-continuous variables from the problem formulation, the authors make use of a number of fundamental results in binary mathematics [57].

As we shall see in Chapter 2, despite its mathematical elegance this methodology is somewhat naïve as it has two important drawbacks. First, it requires the unit commitment formulation to optimize over a set of binary variables augmented by the loss-of-load variables $\psi(\mathcal{C})$. Second, the imposition of an upper bound on either LOLP or ELNS may be problematic when the bound is too strict, a situation which inevitably leads to problems of infeasibility of the underlying electricity market-clearing problem.

On the coupling of short-term and real-time markets

Another important aspect neglected in current electricity market-clearing problems is the coupling between the short-term forward markets (DA and HA) and the real-time market. This coupling is obvious if we consider that the forward markets determine the initial pre-disturbance operation set points carried over to the real-time market [34]. By selecting these set points when clearing a short-term forward market, the system operator may do so on economic grounds only. However, it may be the case that by sacrificing some of the economics in the pre-contingency state, the system operator may save more on average in the course of operation under post-contingency states. This reasoning begs the question whether in running electricity market-clearing problems it is worthwhile optimizing not only over the pre-contingency condition, but also over a pre-selected set of credible post-contingency conditions that may arise in real time. This should have significant implications over the following aspects of market-clearing and operation.

Reserve scheduling and deployment

There is usually a tradeoff that is neglected when system operators schedule reserves services since they never take into account the expected cost of the eventual post-contingency deployment of the reserves. As a result, it may happen that in some instances it is less expensive on average to schedule reserve services with higher capacity costs (the cost for being available to respond to contingencies), but that end up costing less on average when they are deployed. This feature may have significant impacts in unreliable systems wherein reserves are deployed frequently.

Involuntary load shedding

Similar to reserve deployment, the expected costs of any post-contingency involuntary load shedding actions could be evaluated as part of the DA or HA market-clearing processes.

Computational complexity

Considering the coupling between the forward and real-time markets, the resulting market-clearing formulation would require the explicit modeling of both pre- and post-contingency operating conditions. As a result, such a modeling refinement could increase significantly the number of variables and constraints of the short-term market-clearing problem.

1.2.2 Pricing energy and security

A key aspect of any electricity market is its associated pricing mechanism used to charge consumers and to remunerate generators for the energy they respectively consume and produce. There is an ongoing debate on how reserve services should be remunerated and paid for [34, 58–64]. The most common school of thought currently advocates for separate prices for each of the types of reserve services being scheduled [58–60, 63], while recent ideas call for the remuneration of all reserve services scheduled at a bus to be settled at one single price, that of security [34, 64].

In the context where the electricity market-clearing formulation accounts for the uncertainty in the operating conditions, there are a number of pricing issues, especially with respect to the price of security, that are looked at in this dissertation.

1.2.3 Integrating renewable generation resources

With the current trend towards the greater integration of renewable electricity generation resources like wind power into existing grids, research efforts must be devoted to formulate generation scheduling problems taking into account the intrinsic variability and non-dispatchable characteristics of these resources [65, 66]. The type of uncertainty the system operator has to deal with here is definitely different from the uncertainty in the occurrence of equipment failures.

1.2.4 Simplifying security-constrained electricity market-clearing

It is a well-known fact that realistic electricity market-clearing problems are difficult mixed-integer programming problems of very large dimensions. Therefore, adding a security aspect—be it deterministic or stochastic—to market-clearing can render this problem even harder because of the added constraints and variables. Hence, it is of interest to identify ways to simplify security-constrained market-clearing formulations. It is clear, for instance, that one should not attempt to consider the impacts of a contingency if it is known *a priori* that it is covered by one or several more stringent contingencies. It is under this philosophy that this dissertation investigates ways to find these strict contingencies that form an "umbrella" over the other less constraining contingencies, which may be then left out.

1.3 Dissertation Outline

Chapter 2: Market-Clearing With Stochastic Security

This chapter describes in detail the main proposition of this dissertation as it introduces formally the concept of electricity market-clearing with stochastic security. In so doing, we first introduce the model that forms the basis of the stochastic market-clearing proposition, namely the deterministic security-constrained electricity market-clearing problem. Next, we define stochastic security metrics like the expected load not served, and we show how this metric is used in formulating a market-clearing problem with a stochastic security criterion. A discussion of key implementation issues follows. Finally, we report and analyze extensively the results of two case studies where, for instance, we show the economic superiority of stochastic market-clearing over its deterministic counterpart.

Chapter 3: Pricing Under Market-Clearing With Stochastic Security

A theoretical analysis of the marginal prices of energy and security is presented. Specifically, Chapter 3 demonstrates how the Lagrange multipliers associated with contingency scenarios' power balance relations end up determining the marginal values of energy and security at each node of the network. Moreover, allied results relating the use of involuntary load shedding to the values of Lagrange multipliers are derived. All results are found for a general nonlinear market-clearing model and are later specialized for the linear case. A small numerical example based on a two-bus network illustrates some of the results.

Chapter 4: Market-Clearing Under Demand and Wind Generation Uncertainty

In Chapter 4, electricity market-clearing with stochastic security is extended to the problem of day-ahead (or equally hour-ahead) scheduling of power systems with renewable generation resources whose power output cannot be predicted accurately—wind power being the most notorious example. A model for the market-clearing problem is formulated assuming that, without loss of generality, sets of wind power generators form the bulk of the installed intermittent generation capacity. We expose how wind power generation scenarios are built and how their respective probabilities are computed. In addition, we formulate a market-clearing problem general enough to model demand uncertainty as well as bulk energy storage capacity. A small-scale case study is presented. It demonstrates, among other things, how the extra degrees of freedom which are voluntary and involuntary load adjustments can improve the economic efficiency of the electricity market in the presence of intermittent generation resources.

Chapter 5: Umbrella Contingencies in Security-Constrained Market-Clearing

This chapter presents the theoretical and practical aspects associated with the task of identifying "umbrella contingencies" in the context of security-constrained electricity market-clearing problems. Specifically, we present a rigorous definition of the umbrella contingency concept, and we propose a rule to identify and rank them. We then illustrate the validity of the identification and ranking rules for both deterministic and stochastic security-constrained market-clearing problems with the aid of numerical examples.

Chapter 6: Conclusions

This chapter summarizes the main achievements of this investigation. Recommendations for future research are also outlined.

Appendices

A number of appendices complement the exposition of the work. First, in Appendix A, we outline the basic principles used in the calculation of contingency probabilities. Appendix B presents mathematical descriptions of the feasible operating regions of hydrothermal generators as well as demands. These descriptions outline, for example, the constraints affecting generation-side unit commitment decisions and demand-side elasticity limits. Appendix C describes the constraints affecting the determination of both generation- and demand-side reserve levels, while Appendix D shows a proof of the equivalence of two sets of loss-of-load conditions. Next, Appendix E gives the details of the test systems used in the case studies included in the dissertation. Appendix F gives short descriptions of the hardware and software tools used to conduct the numerical studies. Appendix G presents the basic marginal pricing theorem under which energy and security prices are computed and, finally, Appendix H shows how probability distributions of wind power generation prediction errors are discretized.

1.4 Claim of Originality

The following main results of this dissertation can be considered as distinct contributions to knowledge:

1. The most significant original contribution of this dissertation is the theoretical formalization of the concept of electricity market-clearing with stochastic security as presented in Section 2.4 of Chapter 2. This formalization includes the following aspects not previously considered in the formulation of security-constrained electricity market-clearing problems:
   (a) The rigorous definition of security-constrained electricity market-clearing in its deterministic form. This definition is made taking into account a number of practical considerations that include transmission congestion, multi-period unit commitment and the explicit modeling of pre- and post-contingency operating constraints applying to generators, demands and the transmission grid.
   (b) The definition of a stochastic security metric evaluating the expected load not served (ELNS) due to a pre-selected set of random generator and line outages and demand-side disturbances that may happen during any period of the market scheduling horizon.
   (c) This security metric is expressed explicitly in terms of the optimization variables of the market-clearing formulation without the need to define any extra binary variables.
   (d) Since the ELNS stochastic security metric is expressed explicitly in terms of the variables of the market-clearing problem, we propose to use it to specify a stochastic security criterion for the market-clearing problem. The stochastic security criterion proposed is a penalization of the magnitude of the ELNS as weighted by the consumers' value of lost load.
   (e) In addition to the explicit optimization of the expected cost of post-contingency load not served, we propose that the objective function of the stochastic market-clearing problem also co-optimizes the preventive security actions as-

sociated with the pre-contingency state and the corrective security actions associated with the post-contingency states (Proposition 2.1).

2. We derive properties of the marginal nodal prices of energy and security associated with the optimal schedules found by the electricity market-clearing with stochastic security.
   (a) We prove that the Lagrange multiplier of the post-contingency power balance at a given bus is greater than or equal to the expected value of the marginal cost of load shedding if and only if load shedding is applied after that contingency (Proposition 3.1).
   (b) We derive analytical expressions for the expected marginal costs of operation in all pre- and post-contingency states (Propositions 3.2 and 3.3).
   (c) For the special case of linear market-clearing formulations:
      i. We prove that when load shedding is used at a bus after some contingency, it is the last recourse needed by the system operator to balance power (Lemma 3.2).
      ii. We derive analytical expressions for the sensitivities of the optimal level of load shedding to small perturbations in the pre- and post-contingency power balance relations (Corollary 3.2).
      iii. We derive analytical expressions for the Lagrange multipliers associated with post-contingency power balance relations (Theorem 3.1).
3. Based on the principles of electricity market-clearing with stochastic security, we develop a market-clearing formulation that can take into account errors in the day-ahead predictions of intermittent non-dispatchable generation resources, of which we assume are based on wind power. This proposed formulation considers:
   (a) A joint stochastic optimization of hydrothermal generation resources—including multi-period unit commitment—, load demand and involuntary load shedding for given intermittent generation and demand prediction uncertainties.
   (b) Scheduling of large-scale centralized energy storage systems as part of the market-clearing formulation.
4. We define rigorously the notions of umbrella contingencies and of the set of umbrella contingencies for both deterministic and stochastic security-constrained optimal power flow problems (Definition 5.1). From these definitions we further propose:

(a) An identification method for umbrella contingencies based on the Lagrange multiplier vectors of the post-contingency power balance relations (Proposition 5.1).

(b) A heuristic contingency ranking rule based on the contingencies' marginal economic impact on the optimal solution of the market-clearing problem.

Chapter 2

Electricity Market-Clearing With Stochastic Security

I have such foresight as assures success.

The Cenci
Percy Bysshe Shelley, 1792–1822

2.1 Introduction

This chapter introduces the concept of electricity market-clearing with stochastic security, which comes as a counterproposal to the deterministic security-constrained electricity market-clearing problem. Thus, as a first step, we will describe thoroughly the mathematical programming problem corresponding to the deterministic security-constrained market-clearing, formulated here with multi-period unit commitment and transmission constraints.

Next, we present the stochastic security metrics essential in the specification of stochastic security criteria. These metrics quantify either the probabilities or the expected need for involuntary load shedding caused by pre-selected sets of random generator and line outages as well as random demand disturbances. With these stochastic security metrics at hand, we demonstrate how they can be integrated into security-constrained market-clearing schemes. As a consequence of this integration, it is possible, unlike in classical deterministic reserve-constrained unit commitment formulations [22–31], to determine the required levels of reserve services without the use of some rules-of-thumb. Indeed, here the driver for setting the reserve requirements is a rigorous assessment of the expected costs of load not served against those of pre-contingency preventive security actions [41] (reserve scheduling and system pre-positioning) combined with those of post-contingency corrective security actions [41] (reserve deployment).

We then go on to study the behavior of the proposed electricity market-clearing scheme through two cases studies. The first one is based on a small transmission-constrained three-bus network scheduled over a horizon of four hours. In this case, we assess thoroughly the impacts on the resulting generation and reserve schedules of: transmission constraints, generation ramp limits, demand-side reserve, the value of load not served and the constitution of the pre-selected set of contingencies. The second case study looks at the market-clearing results obtained when the IEEE Reliability Test System (IEEE RTS) [67] is scheduled over 24 consecutive hours.

2.2 Security-Constrained Electricity Market-Clearing

As a preliminary step to the introduction of the electricity market-clearing problem with stochastic security, we consider the security-constrained market-clearing (SCMC) problem in its deterministic flavor:[1]

$$\min \left[C_g(\mathbf{u}, \mathbf{g}) + C_r(\mathbf{r}^{up}, \mathbf{r}^{dn}, \tilde{\mathbf{r}}^{up}, \tilde{\mathbf{r}}^{dn}) - B_d(\mathbf{d})\right] \tag{2.1}$$

subject to

Pre-contingency power balance

$$\mathbf{H}(\mathbf{u}, \mathbf{g}, \mathbf{d}, \boldsymbol{\delta}) = \mathbf{0}, \tag{2.2}$$

Pre-contingency transmission line flow limits

$$-\mathbf{f}^{\max} \leq \mathbf{f}(\boldsymbol{\delta}) \leq \mathbf{f}^{\max}. \tag{2.3}$$

[1]We refer the reader to the List of Symbols starting on page xv for the complete listing of the nomenclature.

Moreover, for each of the pre-selected contingencies, $k = 1, \ldots, K$, and all contingency occurrence intervals, $\tau = 1, \ldots, T$, that is for all contingency scenarios (k, τ), the minimization (2.1) is further subjected to

Post-contingency power balance

$$\mathbf{H}(\mathbf{u}(k, \tau), \mathbf{g}(k, \tau), \mathbf{d}(k, \tau), \boldsymbol{\delta}(k, \tau), k, \tau) = \mathbf{0}, \tag{2.4}$$

Post-contingency transmission line flow limits

$$-\mathbf{f}^{\max}(k, \tau) \leq \mathbf{f}(\boldsymbol{\delta}(k, \tau), k, \tau) \leq \mathbf{f}^{\max}(k, \tau), \tag{2.5}$$

Pre- and post-contingency generator constraints

$$(u_{it}, g_{it}, u_{it}(k, \tau), g_{it}(k, \tau), r_{it}^{up}, r_{it}^{dn}, \tilde{r}_{it}^{up}, \tilde{r}_{it}^{dn}) \in \mathcal{G}_{it}; \quad i = 1, \ldots, I, \ t = 1, \ldots, T, \tag{2.6}$$

Pre- and post-contingency demand constraints

$$(d_{mt}, d_{mt}(k, \tau), r_{mt}^{up}, r_{mt}^{dn}) \in \mathcal{D}_{mt}; \quad m = 1, \ldots, M, \ t = 1, \ldots, T. \tag{2.7}$$

The objective of the SCMC, (2.1)–(2.7), is to maximize the total social welfare, or equivalently as seen in (2.1), to minimize the net social cost given the offers for energy and reserve production less the bid-based energy consumption benefits. The energy production cost function, $C_g(\mathbf{u}, \mathbf{g})$, embeds the generators' offered no-load, startup, and variable costs, while the reserve cost function, $C_r(\mathbf{r}^{up}, \mathbf{r}^{dn}, \tilde{\mathbf{r}}^{up}, \tilde{\mathbf{r}}^{dn})$, includes generation- and demand-side offered rates for providing reserves. The demand side of the objective function considers the benefits from energy use in the form of bids for energy consumption, as modeled by the function $B_d(\mathbf{d})$.

Reserve services scheduled here are either of the *spinning* or *non-spinning* type, both of which can be up- or down-going. We assume that if a contingency occurs during some time interval τ, then some of the scheduled reserves may have to be deployed during all subsequent time intervals $t \geq \tau$. In the case of generation-side spinning reserve, be it up-or down-going, it is supplied only by generators already online. On the other hand, non-spinning reserve involves changes in the scheduling status of generators—that is, it involves turning them on or off. For example, a generator already scheduled off could supply up-going non-spinning reserve given that it is free to turn on and to produce some energy during time periods following the occurrence of a contingency. This is unlike down-going non-spinning reserve which can be provided by a generator already online given that it can be brought offline following the contingency occurrence interval τ. In the case of a consumer, the provision of up-going spinning reserve involves being ready to voluntarily decrease its consumption following the contingency occurrence interval τ [21]. For down-going demand-side spinning reserve, consumers that provide this service would be ordered to increase their consumption in the aftermath of a contingency. The details of reserve calculations are addressed thoroughly in Appendix C.

The pre-contingency power balance equality constraints, applying for each network bus and for every period of the scheduling horizon, are represented by (2.2), while the corresponding transmission line flow limits are given in (2.3). We assume in this dissertation that all of the above power flow relations use the linear dc load flow model [32]. The reasons motivating this assumption are twofold. First, our desire here is to demonstrate the practical workings and theoretical soundness of the formulation of the security-constrained electricity market-clearing problem without overloading the exposition with unnecessary technicalities. Second, this assumption does not constitute a departure from current practice as nowadays most electricity market-clearing problems are based on dc load flow models (or ignore the network entirely) in order to limit the problem's computational complexity [23–25, 28, 68].

In the above problem, (2.1)–(2.7), the reserve services are scheduled making sure that the power system can recover, without loss-of-load or line flow limit violations, from the occurrence of any of the credible pre-selected contingencies, $k = 1, \ldots, K$, which may occur during any time interval of the scheduling horizon, $\tau = 1, \ldots, T$. This requirement is expressed mathematically via the post-contingency power balance relations, (2.4), as well as by the post-contingency transmission line flow constraints, (2.5).

Unlike the use of the dc load flow model, we point out that this approach truly constitutes a departure from current practice in formulating security-constrained market-clearing problems [22–31]. However, as justified in [34] and [64], this generalization is necessary in situations when transmission congestion may be present in the pre- and post-contingency states, and when the market-clearing solution has to consider the costs of deploying post-contingency actions. This formulation is advantageous because it permits the definition of all types of reserve from the differences between the pre- and post-contingency operating states, and, more fundamentally, that the area reserve criteria do not have to be defined *a priori* using some rule-of-thumb [22–24, 26–31, 35]. Rather, the reserve criteria are *implicit* in the requirement that power must balance at every node for all pre-specified post-contingency states, while satisfying all the operational requirements of the transmission grid, including those of the generators and the consumers.

The sets of constraints applying to the generators, $\mathcal{G}_{it}$ in (2.6), represent all operational limitations for all pre- and post-contingency states over the scheduling horizon. These operational limitations include the classical unit commitment constraints applying to hydrothermal generators such as minimum up- and down-times, ramping, minimum and maximum output power [45, 68–80], as well as reserve levels. Appendix B provides a detailed description of minimum up- and down-time, ramping as well as capacity constraints based on the computationally-efficient Ruiz-Peinado unit commitment formulation [79], while Appendix C describes in details the workings of the reserve constraints. On the demand side, the operating sets $\mathcal{D}_{mt}$ appearing in (2.7) describe operational restrictions such as elasticity limits, further explained in Section B.3 of Appendix B, and demand-side spinning reserve limits, which are detailed in Section C.1.2 of Appendix C.

2.3 Stochastic Security Metrics

The deterministic SCMC problem just formulated has two main weaknesses. The first one is that if *involuntary* load shedding—as opposed to *voluntary* demand adjustments in the form of demand-side reserve [21]—is not permitted in post-contingency states it may be impossible balance power at every bus while satisfying simultaneously the entire set of transmission line flow limits (2.5) and the generator and demand operational constraints (2.6) and (2.7) respectively. The second drawback of the deterministic approach is that in instances when the market-clearing problem is feasible without the use of involuntary load shedding, it is necessary to cover all contingencies regardless of their likelihood of occurrence. By neglecting the probability of the occurrence of contingencies, the power system suffers from an increase in its overall *expected* social costs and, particularly as will be shown later in Section 2.5, this overly conservative requirement may lead to increases in the marginal costs—and thus in the prices—of security and energy.

These two drawbacks give a strong motive to formulate an electricity market-clearing problem with a stochastic security criterion that will take into account the likelihood of the contingencies and that will permit the use of involuntary load shedding only when justified economically or for feasibility reasons. We reiterate here that in this dissertation load shedding is meant to be involuntary load shedding as opposed to voluntary demand reduction offered to the market as up-spinning reserve. Therefore, under the proposed stochastic SCMC formulation, we are ready to tolerate some amounts of involuntary load shedding if the contingencies causing these loss-of-load events happen with sufficiently low probabilities and if their corresponding increases in the expected social cost are small.

2.3.1 Loss-of-load calculations

In the proposed formulation, the amount of involuntary load shed at bus m during period t due to contingency k occurring during interval τ, denoted by $l_{mt}(k,\tau)$, is computed from the post-contingency power balance relation

$$l_{mt}(k,\tau) = d_{mt}(k,\tau) + \sum_{\substack{\ell \in \mathcal{B}_m \\ \ell \notin \mathcal{C}_k}} f_\ell(\boldsymbol{\delta}_t(k,\tau), k, \tau) - \sum_{\substack{i \in \mathcal{A}_m \\ i \notin \mathcal{C}_k}} g_{it}(k,\tau), \tag{2.8}$$

where $d_{mt}(k,\tau)$ is the demand at bus m, $f_\ell(\boldsymbol{\delta}_t(k,\tau), k, \tau)$ is the power flow in line ℓ, $\boldsymbol{\delta}_t(k,\tau)$ is the vector of voltage angles and $g_{it}(k,\tau)$ is the generation level of unit i. The expression in (2.8) applies for any single or compound outages and load disturbances. The latter can be made up of the concurrent failure of some lines and some generators, combined with some pre-specified load disruptions, as specified in the set of failed/disrupted components $\mathcal{C}_k$.

In addition, involuntary load shedding cannot be negative, and it cannot be greater than the actual load

$$0 \leq l_{mt}(k,\tau) \leq d_{mt}(k,\tau). \tag{2.9}$$

This applies for all contingencies $k = 1, \ldots, K$, buses $m = 1, \ldots, M$ and time periods that follow the occurrence of contingencies, that is $t \geq \tau$.

We further remark that it is needed to impose that involuntary load shedding should not be applied in the pre-contingency state; in other words, we should impose that $l_{mt}(k,\tau) = 0$

if $t < \tau$. Note, however, that in order to meet this condition, we assume that there is sufficient generation and transmission capacity available in the pre-contingency state to be able to meet the consumers' demands.

To determine the expected extent of involuntary load shedding, we define the quantity $\hat{l}_{mt}$, the expected load not served (ELNS) [46, 50] at bus m during period t averaged over all contingencies occurring randomly over the past intervals

$$\hat{l}_{mt} = \sum_{k=1}^{K} \sum_{\tau=1}^{t} p(k, \tau) l_{mt}(k, \tau) \Delta. \tag{2.10}$$

In (2.10), the quantity $p(k, \tau)$, being the probability of the event "*no contingencies occur during the scheduling horizon except for contingency k, which occurs during interval τ*," is calculated from historical mean time to failure data (assumed to remain constant over the scheduling horizon) as shown in Appendix A.2. In addition, the quantity Δ represents the time duration (generally in hours) between two successive periods t and $t + 1$.[2]

2.3.2 Stochastic metrics

As mentioned in Chapter 1, the ELNS is not the only stochastic metric available to assess the risk and/or the expected consequences of involuntary load shedding. The loss-of-load probability (LOLP) [46, 50], measuring the probability that load shedding events happen following the occurrence of contingencies, is another valid stochastic security metric assessing the risk of load shedding. Contrary to the ELNS, however, the LOLP does not give any information regarding the relative importance of the contingencies, as it fails to give a physical measure of the "damage done"—*i.e.* the energy not supplied. This leads to the second argument against the use of the LOLP to impose a stochastic security criterion. System operators generally have more facility in dealing with physical quantities (power or energy, for instance) than in treating dimensionless numbers like probabilities. Moreover, as shown before in Chapter 1 and in [50, 53], the computation of the LOLP as part of the market-clearing process requires the definition of extra binary variables (ψ) to indicate the occurrence of loss-of-load events. We recall, though, that in Chapter 1 and [50] the ELNS is also computed using extra binary variables; however, we show in Appendix D

[2] We remark here that this assumes that the duration of the loss-of-load event for a given period t lasts Δ units of time and that the "rates" of load shedding, $l_{mt}(k, \tau)$, are assumed constant over that period.

that in the proposed market-clearing formulation, the ELNS can be equivalently computed without these extra binary variables. Thus, the LOLP is also inferior to the ELNS from a computational complexity point of view under the assumption that good modeling practice in mixed-integer optimization generally seeks to reduce the number of binary variables as much as possible [57, 81–84]. Lastly, we might have considered as well the loss-of-load expectation (LOLE) [46], evaluating the expected number hours during which loss-of-load events could occur. However, like the LOLP, it requires the definition of extra binary variables for its computation; furthermore, the LOLE fails to give a good measure of the severity of the contingencies. There is no doubt in our opinion that the ELNS is by far a superior security assessment tool and, as a result, it will be the sole stochastic security metric used in the remainder of this dissertation.

2.3.3 Stochastic security criteria

There are three ways to impose a stochastic security criterion based on the ELNS. The first one, as proposed initially in Chapter 1 and in [50], is to impose an upper limit on the magnitude of the ELNS calculated either bus-by-bus, area-by-area or over the whole system. These upper limits can be imposed also over subsets of time periods of the scheduling horizon (for example, hour-by-hour, over on-peak and off-peak hours, or simply over the entire scheduling horizon). The second way is to add a penalty function, increasing monotonically with ELNS, to the objective function of the market-clearing problem. Lastly, the third method involves combining both upper bounds and a penalty function.

Imposing bounds on the ELNS as part of or as the sole stochastic security criterion is somewhat disadvantageous. One difficulty with this approach is the specification of the bounds' upper limits. Most likely, these would need to be specified by a regulatory agency. This task would probably be a difficult process since the rules underlying this specification may be hard to justify and could lead to some consumer inequities. Moreover, even though stochastic market-clearing based on upper bounds on the ELNS permits load shedding—unlike the deterministic approach outlined before—, there are still possibilities that the specified upper bounds on ELNS cannot be satisfied when there are insufficient reserve resources or transmission capacity available, as well as in cases when the system resources are fairly unreliable. Lastly, under the ELNS bounding approach, involuntary load shedding would be applied until the ELNS inequalities became binding as a way to minimize the social cost of scheduling and deploying reserves despite the possible fact that there may be sufficient reserve resources and transmission capacity available to cover the set of pre-selected contingencies. Such a situation, in general, would be unacceptable to the consumers.

With the explicit goal of avoiding these shortcomings, in the next section, we propose to use a stochastic security criterion only based on the penalization of ELNS within the objective function.

2.4 Electricity Market-Clearing With Stochastic Security

This section presents the kernel of the proposed electricity market-clearing with stochastic security. First, we describe how this problem is formulated as a two-stage stochastic optimization problem with fixed recourse [85], wherein the uncertainty affects both the objective function and its constraints. We then follow with a formal discussion of the proposal.

2.4.1 Formulation

Assumptions

For the proposed formulation, as for the deterministic SCMC presented before, the following assumptions are made:

1. Only single contingencies may occur over the length of the scheduling horizon, where, as previously mentioned, such a contingency could be a simultaneous compounded failure; and,
2. The reserve service requirements are left undetermined in the post-contingency states because their specification and optimization would necessitate taking into account non-simultaneous, sequential contingencies.

In relation with the second assumption, we note that the important increase in the complexity of the problem (both in terms of the number of variables and constraints) associated with the inclusion of such low probability events may not be warranted for most practical situations. In fact, following the occurrence of any contingency, it is implied that the system operator has to assess the state of the power system. This calls for a full reevaluation of the levels of reserves required to respond to further contingencies. We note that the system operator can well use the proposed formulation (or some simplified version of it) to accomplish this task.

Objective

The objective function of the electricity market-clearing with stochastic security is the keystone of the global proposal of this chapter; we introduce it formally in the following proposition.

Proposition 2.1 (Objective of electricity market-clearing with stochastic security). *The objective of electricity market-clearing with stochastic security is to schedule the pre- and post-contingency generation and consumption as well as reserves and involuntary load shedding minimizing the total expected social cost, that is*

$$\min p(0)\left[C_g(\mathbf{u},\mathbf{g}) + C_r(\mathbf{r}^{up},\mathbf{r}^{dn},\tilde{\mathbf{r}}^{up},\tilde{\mathbf{r}}^{dn}) - B_d(\mathbf{d})\right] + \sum_{k=1}^{K}\sum_{\tau=1}^{T} p(k,\tau)\left[C_g(\mathbf{u}(k,\tau),\mathbf{g}(k,\tau)) - B_d(\mathbf{d}(k,\tau))\right] + \sum_{m=1}^{M}\sum_{t=1}^{T} v_{mt}\hat{l}_{mt}. \quad (2.11)$$

The market-clearing objective (2.11) *consists of the sum of three terms: (i) the expected social cost associated with the pre-contingency state [happening with probability* $p(0)$*; see Appendix A.2] that accounts for the social cost of scheduling pre-contingency generation, load and reserve services; (ii) the expected social cost associated with the post-contingency decisions that comprise the deployment of reserves that may involve the possible switching on or off of generators as well as the re-dispatching of generation and consumption; and, (iii) the expected cost of involuntary load shedding, for which the quantity* v_{mt} *is the value of lost load (VOLL) [10, 14, 86–90] at bus* m *during period* t.

Constraints

Like the deterministic market-clearing problem described before in (2.1)–(2.7), the proposed stochastic minimization problem is constrained by the same pre-contingency power balance and transmission line flow limits [respectively (2.2) and (2.3)], the same post-contingency transmission line flow limits (2.5), and the same generation and demand constraints [(2.6) and (2.7) respectively]. The differences between the two formulations, in addition to the objective function, lie in the post-contingency power balance constraints, which now take into account the involuntary load shedding variables, $\mathbf{l}(k, \tau)$

$$\mathbf{H}(\mathbf{u}(k,\tau), \mathbf{g}(k,\tau), \mathbf{d}(k,\tau), \boldsymbol{\delta}(k,\tau), \mathbf{l}(k,\tau), k, \tau) = \mathbf{0}. \tag{2.12}$$

Moreover, the stochastic problem ought to meet the load shedding bounds already defined before in (2.9).

In the field of stochastic optimization, the optimization problem proposed here is called a *scenario analysis* problem [85, 91, 92]. Here we define a scenario as:

Definition 2.1 (Scenario). A *scenario* describes a possible outcome of a randomly-varying state of nature. Here, the realizations of the randomly-varying states of nature, which define the scenarios identified by the pairs (k, τ), are the specific occurrences of contingencies $k = 1, \dots, K$ during one of the time periods $\tau = 1, \dots, T$, as well as the pre-contingency state.

Typical of scenario analysis problems are *bundle constraints* [91]. Here, these constrains need to be added to the sets describing the operational limits of the generators and demands, $\mathcal{G}_{it}$ and $\mathcal{D}_{mt}$ respectively, to model the nonanticipatory character of the schedules. Similar comments apply to the network-related variables and functions, $\boldsymbol{\delta}$ and $\mathbf{f}(\boldsymbol{\delta})$. In plain words, bundle constraints impose the condition that as long as the uncertainty has not been resolved, optimal operational schedules should follow the schedule associated with the pre-contingency scenario. Mathematically, this translates to the requirement that before the occurrence of contingency k during the interval τ, the following conditions must hold

$$u_{it}(k,\tau) = u_{it}; \qquad i = 1,\dots,I,\ t < \tau, \tag{2.13}$$

$$g_{it}(k,\tau) = g_{it}; \qquad i = 1,\dots,I,\ t < \tau, \tag{2.14}$$

$$d_{mt}(k,\tau) = d_{mt}; \qquad m = 1,\dots,M,\ t < \tau, \tag{2.15}$$

$$l_{mt}(k,\tau) = 0; \qquad m = 1,\dots,M,\ t < \tau, \tag{2.16}$$

$$\delta_{mt}(k,\tau) = \delta_{mt}; \qquad m = 1,\dots,M,\ t < \tau, \tag{2.17}$$

$$f_\ell(\boldsymbol{\delta}_t(k,\tau), k, \tau) = f_\ell(\boldsymbol{\delta}_t); \qquad \ell = 1,\dots,L,\ t < \tau, \tag{2.18}$$

for each $t = 1, \dots, T$.

After contingency k, which occurred during the time interval τ, all post-contingency variables should keep on satisfying the constraints applicable in the pre-contingency state, with the exception of those variables associated with the failed elements defining the specific contingency. For example, given that contingency k leads to the loss of generator j—that is, $j \in \mathcal{C}_k$—, the post-contingency generator variables must satisfy still the constraint $(u_{it}(k,\tau), g_{it}(k,\tau)) \in \mathcal{G}_{it}$, given that $i \notin \mathcal{C}_k$. For generator j, however, we have $(u_{jt}(k,\tau), g_{jt}(k,\tau)) = (0,0)$.

Lastly, we reiterate—and we demonstrate formally in Appendix D—that in calculating the ELNS, constraints (2.8), (2.9) and the action of the market-clearing objective (2.11), yield ELNS values equivalent to those found through the conditions developed in Chapter 1 and [50] which required the definition of extra binary variables. However, if one uses the LOLP or the LOLE as the stochastic security metric, the use of these extra binary variables is necessary.

Stochastic programming classification of electricity market-clearing with stochastic security

Birge and Louveau [85] describe the general (linear) form of the two-stage stochastic programming problem with fixed recourse as the following optimization problem:

$$\min_{\mathbf{x}\geq \mathbf{0}, \mathbf{y}(\omega)\geq \mathbf{0}} \mathbf{c}^T\mathbf{x} + E_\omega\left[(\mathbf{q}(\omega))^T\mathbf{y}(\omega)\right] \tag{2.19}$$

subject to

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{2.20}$$

$$\mathbf{T}(\omega)\mathbf{x} + \mathbf{W}(\omega)\mathbf{y}(\omega) = \mathbf{h}(\omega). \tag{2.21}$$

Here, $\omega \in \Omega$ is a random variable (RV) having some known probability distribution over the support space $\Omega \subset \mathbb{R}$, and $E_\omega[\cdot]$ is the mathematical expectation operator over the RV ω. In (2.19)–(2.21), the variables contained in the vector $\mathbf{x}$ are known as the *first stage* variables, which model the decision-making process done prior to the resolution of the uncertainty, while the vector $\mathbf{y}(\omega)$ contains the *second stage* variables which are dependent on the specific realization of the RV ω. In addition, the general problem formulation shows dependence on the specific realization of the RV ω in: (i) the objective function [through the vector $\mathbf{q}(\omega)$]; and, (ii) the constraints [through the matrices $\mathbf{T}(\omega)$ and $\mathbf{W}(\omega)$ and the right-hand side vector $\mathbf{h}(\omega)$].

It is clear by inspection of (2.11) that the objective function of the proposed electricity market-clearing with stochastic security is a specific instance of (2.19), wherein the randomness component lies in the realization of one of the probable contingency scenarios. In this case, the objective function optimizes only over the second-stage variables, which are dependent on the realization of a given contingency scenario. Likewise, all the constraints depend on the realization of some given contingency scenario, as indicated by the arguments (k, τ).

We should point out that variants of the formulation of the electricity market-clearing with stochastic security may be constructed. In those, for instance, subsets of the variables associated with the pre-contingency scenario—which has a probability of occurrence below one—may be considered as part of the set of first-stage decision variables. This is the case, as will be seen in Chapter 4, when unit commitment decisions are made ahead of time and cannot be altered once the uncertainty is revealed. Likewise, since reserve levels are not necessarily updated when a given scenario is reached (as assumed here), these may be treated as first-stage variables with a corresponding cost of supply, $C_r(\mathbf{r}^{up}, \mathbf{r}^{dn}, \tilde{\mathbf{r}}^{up}, \tilde{\mathbf{r}}^{dn})$, incurred with unit probability.

We recall also that stochastic optimization methods have other very important applications in the field of generation planning problems. For completeness, next, we provide some of the most important applications of stochastic optimization to the field of generation scheduling.

Hydroelectric generation operations planning

Hydroelectric generation planning is a well-known challenging stochastic optimization problem. Generally, its objective is to maximize the expected value of the water stored in reservoirs contingent on the realization of randomly-varying electricity demand and water inflows. This problem has very large dimensions due to the variety of scenarios that must be considered. It is also computationally demanding because of complex river basin flow coupling constraints and nonlinear hydro turbines' efficiency characteristics. The work of the Brazilians Pereira and Pinto is probably the most famous contribution in this field [93, 94]. This work is remarkable for its rigourous hydroelectric operations planning formulation and for the development of a solution method by means of Benders' decomposition.

Generation operations planning under demand uncertainty

The second classical application of stochastic optimization in electricity generation planning is the problem of scheduling in the presence of demand uncertainty. In most instances, scenario analysis type problems are formulated with the objective of minimizing the expected generation cost such that the power should balance *almost surely* for the predicted, but uncertain, load [92, 95, 96]. Other formulations use a "chance constrained" approach whereby the probability that it will be possible to meet the randomly-varying load is constrained to be above some lower limit [97].

Generation operations planning under price uncertainty

In recent years, electricity restructuring has spurred interest in price-driven generator self-scheduling methods [77, 78]. Obviously, given that price predictions for electricity and/or fuels are uncertain, stochastic optimization methods are well suited to tackle these problems. In general, such problems attempt to optimize expected profits and/or some risk measures of a given generation company subject to the operating constraints of its generation capacity portfolio and its set of pre-agreed bilateral contracts. This problem, unlike the problem considered in this dissertation and the two other problems just described, does not have to worry about the complicating constraints like nodal power balance relations or transmission line flow limitations. A vast body of literature on this problem is now available; see, for example, the works of [98–105].

2.4.2 Discussion

On the innovative features of electricity market-clearing with stochastic security

The proposed electricity market-clearing formulation provides a number of key generalizations in comparison to earlier security-constrained market-clearing schemes. First, the work pioneered by Monticelli *et al.* [106] on optimal post-contingency corrective control actions in security-constrained optimal power flow problems did not consider unit commitment decisions, contingency probabilities, nor the expected social costs of corrective actions—like generation re-dispatch and involuntary load shedding. Likewise, the works of [86, 87, 107, 108] take into consideration the expected costs of preventive and corrective security actions, but again only for optimal power flow problems without unit commitment.

The intentions of the authors of [86, 87, 107] were to develop methods to compute the expected total and marginal social costs of security measures for pricing purposes, whereas Kimball *et al.* [108] essentially concern themselves with the development of a decomposition method to solve a specific instance of a stochastic optimal power flow problem. Wang *et al.* [109] propose a risk-based cost/benefit approach for the scheduling of reserve services to compensate for generator failures. However, as their method assumes some *a priori* pre-contingency generation schedule, these authors fail to recognize the tight coupling that exists between the energy and the reserve scheduling tasks. It is well known that such functional separation can lead to inefficient or infeasible market-clearing results [34, 110]. Rashidinejad *et al.* [111] contemplated the idea of simultaneously accounting for the costs of reserve scheduling and deployment. Nonetheless, they based their reserve-constrained market-clearing problem formulation on simple rule-of-thumb reserve criteria—for example setting the requirement equal to a fraction of the demand or the largest online generator—, while they ignored unit commitment, transmission network congestion as well as the time dynamics associated with power system operations. The work of Carpentier *et al.* [112] done at Électricité de France in 1996 shares some common points with the proposed approach; for instance, the authors consider generator failure scenarios and the optimization of post-contingency generation as part of the scenarios. However, they did not include a network model in their formulation nor did they consider the cost to the consumers corresponding to involuntary load shedding events. For these authors, any generation deficit does not necessarily lead to load shedding; they claim that their power imbalance penalty represents the cost of extraordinary measures including the use of resources outside of the utility's service territory or the startup of expensive peaking generators. In addition, unlike here, the unit commitment problem is formulated as a nonlinear mixed-integer programming problem, a feature that increases computational complexity and that reduces the likelihood of finding a global optimal solution.

Arroyo and Galiana in [64] formulated a security-constrained market-clearing tool based on a single-period network-constrained unit commitment; however, they did not consider the probabilistic features of the proposed formulation, the cost of deploying corrective actions nor involuntary load shedding. Many security-constrained unit commitment formulations have been proposed in the literature and are in use; this is the case in PJM, New York ISO, ISO New England as well as in New Zealand and Ontario [22–31]. None of these security-constrained market-clearing formulations, however, consider explicitly how the probability of occurrence of contingencies and their associated post-contingency corrective actions can affect generation, demand and reserve schedules. Furthermore, the works reported in the literature have not treated jointly the complicating aspects of electricity market-clearing that we consider here, namely: (i) multi-period unit commitment; (ii) probabilistic security criteria based on the expected value of involuntary load shedding; (iii) pre- and post-contingency transmission line flow limits; and, (iv) the expected value of the social costs of *both* preventive and corrective security actions. Most importantly, we point out that none of the works cited above intended to provide the necessary theoretical foundations nor the implementation details of an electricity market-clearing mechanism based on a stochastic security criterion as it is done in this dissertation.

On the integration of electricity market-clearing with stochastic security in the wider power system operations paradigm

A practical implementation of the proposed electricity market-clearing scheme is certainly tied to the appropriate functioning of fast emergency actions that are automatically implemented in the immediate aftermath of a disturbance. In fact, the sets of preventive and corrective actions determined by the proposed market-clearing scheme are by themselves not sufficient to prevent cascaded outages that could lead to a complete power system collapse. The assumption here is that the fast emergency actions, taking place in the order of fractions of seconds to a few minutes following the occurrence of a contingency, prevent the system from drifting away from its secure pre-contingency operating point into instability. For most situations, the fast and local automatic emergency actions based on primary frequency regulation [32–34, 36] are sufficient to do so. In cases of more severe contingencies, however, automatic involuntary load shedding actions, based on frequency and voltage-sensitive relaying, may have to complement the primary frequency regulation actions.[3]

The post-contingency corrective actions determined by the market-clearing scheme belong to the realm of tertiary regulation actions implemented 10–20 minutes after a disturbance [33, 34]. In resolving the market-clearing problem, the system operator centrally

[3]The automatic involuntary load shedding actions to which we refer here are *not* of the same type as those calculated by the market-clearing scheme. The economics behind these load shedding actions are ignored as they fail to discriminate between consumers having high or low value of lost load. The unique goal of these automatic actions is to keep the system from collapsing.

determines how the power system can be re-positioned during the tertiary regulation interval so as to meet all its technical limits and to operate more economically, two aspects not addressed by the previous primary regulation interval.

One obvious advantage of the proposed market-clearing is the co-optimization of the pre- and post-contingency states. By so doing, the system operator has the possibility of pre-positioning the system, through preventive control actions, in such a way that it may be better prepared to react to those highly severe and probable contingencies. Moreover, the explicit modeling of the post-contingency states allows for a better management of corrective control actions. As the expected costs of reserve scheduling and deployment and those of involuntary load shedding are calculated jointly, the system operator can generate monetary savings in terms of the net social cost as well as in the marginal prices of energy and security. These features will be illustrated in Section 2.5 and further analyzed in Chapter 3. We must note also that so far none of the existing electricity markets worldwide has implemented this kind of functionality.

On the use of involuntary load shedding

Historically, power systems have been operated with the implicit requirement that load be shed as a last recourse only, regardless of the cost associated with the deployment of the available resources needed to counter the disturbances. Thus, the explicit consideration of involuntary load shedding as a potential corrective post-contingency action in a market-clearing scheme, as we propose here, is unorthodox. However, one has to assume that involuntary load shedding should be used sparingly because in most situations it is much more costly to cut load than to have to schedule and deploy reserves. In addition, one weakness here is the fact that the value of involuntary lost load at a given bus and time, as specified by v_{mt}, is uncertain and surely depends on the duration of a given interruption [14, 46]. As of now, it is still unclear whether it should be specified by a regulatory body or the load-serving entities. Stoft [10], as well as Joskow and Tirole [14] provide insightful theoretical economic discussions on this subject. We believe that, for questions of social fairness, regulators are in better positions to determine a uniform value of lost load across an entire or some areas of a given network. For example, the Australian regulatory authorities have adopted a uniform value of lost load of the order of AU\$10 000 per megawatt-hour [10]. Of course, buses feeding critical loads like hospitals and airports, for example, should be assigned very high values of v_{mt}. Nonetheless, one should be aware that in situations when there is insufficient transmission capability following some contingency (in cases of network separation, for example), even critical loads may have to cut down their consumption regardless of their level of v_{mt}.

We should stress also that there has been, and there are still nowadays vast industry-driven research efforts devoted to the evaluation of social costs of power interruptions; see, for example, the econometric and engineering studies reported in [10, 14, 46, 88, 113, 114]. These efforts must be continued and should evolve in the direction set by the constraints and characteristics of restructured power systems wherein the notion of "value" to the consumer is fundamental.

On the computational complexity of electricity market-clearing with stochastic security

One may correctly infer that optimizing over the post-contingency on/off generator status variables, $u_{it}(k, \tau)$, is computationally costly (both in terms of core memory usage and CPU time) because the number of possible binary variable combinations grow exponentially with the number of contingencies and the possible contingency occurrence times over the scheduling horizon. Therefore, in cases where large-scale systems need to be scheduled, it may be necessary to forgo the explicit optimization of post-contingency binary variables by fixing them to their pre-contingency levels. This is done, simply, by requiring that $u_{it}(k, \tau) = u_{it}$ for all contingencies k and times of occurrence τ. We note that, by so doing, non-spinning reserves are no longer defined in the formulation.

Likewise, since both the pre- and post-contingency variables and constraints are explicitly modeled, the dimensions of the optimization problem that may need to be solved in realistic systems can be quite significant. The consideration of solution techniques based on Benders' decomposition [27,31,82,85,112,115–117] is a promising way to deal with large systems by limiting the size of the constraint set stored in core memory and by taking advantage of the parallelism offered by this solution strategy. One can make similar comments about the "branch-and-price" decomposition technique [118–120]. In addition, several authors [105, 121–123] have proposed and used some rigorous scenario reduction algorithms, where via a pre-processing step, these algorithms aggregate the scenarios having similar impacts in a given stochastic optimization problem. In a variant of scenario reduction, one seeks to eliminate the need to consider the variables and constraints associated with scenarios that lie under the "umbrella" of other more constraining scenarios. With that goal in mind, Chapter 5 proposes a technique to discover those umbrella contingencies.

2.5 Case Studies

In this section, we analyze the market-clearing formulation with stochastic security just developed through two case studies solved using mixed-integer linear programming (MILP) techniques.

First, Section 2.5.1 studies a small-scale system. Although this case is simple enough so as to verify readily the validity of its outcomes, it illustrates well many of the ramifications of electricity market-clearing with stochastic security. Specifically, we analyze: (i) the effects of transmission line flow limits; (ii) the opportunity costs of excluding non-spinning reserve; (iii) the impacts of demand-side valuation of energy not served; (iv) the influence of ramping limits; and, (v) the effects of the pre-selected set of contingencies. We also compare the outcomes of the proposed stochastic market-clearing to those of its deterministic counterpart described in Section 2.2. In addition, we derive and compare the nodal prices of energy and security associated with the stochastic and the deterministic market-clearing optimal schedules.

Then in Section 2.5.2, we apply the proposed market-clearing scheme to the IEEE Reliability Test System [67] which is scheduled over a 24-hour horizon. This case sheds some light on the dimensionality issues of the proposed market-clearing scheme.

2.5.1 Small-scale study

This case study analyzes the scheduling of System A, described in Appendix E.1. Here, we assume that the set of pre-selected contingencies characterizing the security criterion include all single generator and line outages. We index the failures of generators 1, 2 and 3 with $k = 1, 2, 3$ respectively, while $k = 4, 5, 6$ respectively index the failures of lines 1, 2 and 3. We moreover assume that any one of these six contingencies can occur during any of the four hours of the scheduling horizon, that is $\tau = 1, \ldots, 4$.

We remark that the $6 \times 4 = 24$ contingency scenarios characterizing the security criterion here are few compared to the actual number of possible contingency scenarios. The latter is defined by all possible combinations of generator and line outages over all possible times of failures, including sequential failures, which equals $\sum_{n=1}^{6} 4^n \binom{6}{n} = 15\,624$. Since time periods last one hour each, the parameter Δ is assumed to be one hour-long.

The formulation of the electricity market-clearing problem is already in linear form given the system data and the objective function described in Appendix E.1. The resulting mixed-integer linear problem was then solved using CPLEX 9.0.0 under GAMS on *Paco* (CPLEX, GAMS and *Paco* are described in detail in Appendix F). For all the subcases shown below, computation times (CPU times) were all below one second.

Results and analysis

Tables 2.1 and 2.2 summarize the main features of the optimal schedule obtained for System A. First, Table 2.1 gives the breakdown of the corresponding optimal expected social cost. Without much surprise, the component corresponding to the expected operating cost under the pre-contingency state dominates with 95.01% of the total amount. The remaining portion of the expected cost is incurred following the occurrence of the pre-selected contingencies. This amount splits between the expected cost of deploying the reserves by re-dispatching and turning on or off generators (3.30% of the total) and the expected cost assumed by the consumers because of the use of involuntary load shedding (1.69% of the total).

Table 2.1 Breakdown of expected social costs—System A

	Total	Pre-contingency	Reserve deployment	Loss-of-load
Cost ($)	7228.74	6868.15	238.51	122.07
% total cost	100.00	95.01	3.30	1.69

This breakdown agrees with the probability level of the event that none of the pre-selected contingencies occurs over the four-hour scheduling horizon, $p(0) = 0.9673$. We observe also that this probability is somewhat close to its associated expected cost proportion (95.01%). In the same vein, the optimal expected cost proportions associated with reserve deployment and involuntary load shedding (3.30% + 1.69% = 4.99%) are of the same order of magnitude as the probabilities of the contingency scenarios put together, that is $\sum_{k,\tau} p(k,\tau) = 0.0323$.

Table 2.2 summarizes the optimal generation and reserve schedules of the generators as well as the demand-side reserve contributions associated with the pre-contingency scenario. In this case, being incrementally the least expensive at \$20 per megawatt-hour, generator 3 supplies energy over all four hours; however, it does not provide any reserve service. Being the next least expensive unit available to provide energy at \$30 per megawatt-hour, generator 1 generates the demand not yet fulfilled by generator 3 during the higher load hours 2 and 3. Unlike generator 3, generator 1 supplies non-spinning up reserve during hour 4, up-spinning reserve during hour 2, and down-spinning reserve during hour 3. Lastly, generator 2, which offers energy at the highest rate among all three units at \$40 per megawatt-hour, is not committed, but nonetheless supplies some non-spinning up reserve during hours 2 and 3.

On the side of the demand at bus 3, Table 2.2 shows that it does not get to provide any reserve based on voluntary load adjustments. The last row of Table 2.2, however, indicates that the optimum market-clearing schedule calls for some involuntary load shedding during periods 1 and 3. An observation of that same row indicates that since the value of lost load is high (\$1000 per megawatt-hour), the ELNS at bus 3 calculated over the four hour-long horizon is low—that is, $\sum_t \hat{l}_{3t} = 122.1$ kWh or 0.05% of the 260 MWh of energy scheduled to be consumed over the whole horizon.

We observe that even though involuntary load shedding is costly, the market-clearing solution still calls for some amount of it. This result displays the essence of the stochastic security criterion that simultaneously assesses the credibility and the severity of the contingencies making up that criterion.

Below, still referring to Table 2.2, we study how the different reserve levels are set for each of the four hours, paying particular attention to the reasons why load is shed under market-clearing with stochastic security.

Hour $t = 1$

We observe that during this hour there are no reserve services being scheduled, and that, consequently, the failure of generator 3 would lead to loss-of-load. Such scheduling behavior is surely unique to market-clearing with stochastic security. This reflects the fact that the loss of generator 3 has both a low enough probability combined with a low enough load shedding impact in comparison to the expected costs of reserve scheduling and deployment.

Table 2.2 Pre-contingency generation, reserves and ELNS—System A

		Time t (h)			
		1	2	3	4
g_{1t}	(MW)	0.0	30.0	60.0	0.0
r_{1t}^{up}	(MW)	0.0	20.0	0.0	0.0
r_{1t}^{dn}	(MW)	0.0	0.0	5.0	0.0
$\tilde{r}_{1t}^{up}$	(MW)	0.0	0.0	0.0	40.0
$\tilde{r}_{1t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{2t}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{2t}^{up}$	(MW)	0.0	30.0	60.0	0.0
$\tilde{r}_{2t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{3t}	(MW)	30.0	50.0	50.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{up}$	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
d_{3t}	(MW)	30.0	80.0	110.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_{3t}$	(kWh)	117.7	0.0	4.4	0.0

It is simple to show by trial-and-error that this contingency gets fully covered by reserve only if the value of lost load is raised over and above \$1171 per megawatt-hour for that hour. We refer the interested reader to Chapter 3 for a detailed theoretical analysis of the observed behavior. Furthermore, it is obvious from Table 2.2 that failures involving any of the other generators or lines do not cause loss-of-load during that hour.

Hour $t = 2$

Here, the loss of generator 1 that may occur during one of the hours $\tau \in \{1, 2\}$ is fully covered by the 30 megawatts of non-spinning up reserve supplied by generator 2. Likewise, the failure of generator 3 is taken care of by the 20 megawatts of up-spinning reserve provided by generator 1 combined with the 30 megawatts of non-spinning up reserve from generator 2. By inspection, no line loss occurring during $\tau \in \{1, 2\}$ can cause loss-of-load and neither can the failure of generator 2.

Hour $t = 3$

The failure of generator 1 happening during one of $\tau \in \{1, 2, 3\}$ is covered in full by the 60 megawatts of non-spinning up reserve provided by generator 2. In cases when generator 3 fails during one of $\tau \in \{1, 2, 3\}$, generator 1 is required to back down its generation level by 5 megawatts (explaining why it provides 5 megawatts of down-spinning reserve) at the same time when generator 2 has to turn on to generate 55 megawatts, deploying its provision of non-spinning up reserve. Accounting all these reserve deployment actions, we find out that the involuntary load loss associated with this contingency is nil for that hour. The post-contingency dispatch here maximizes the use of the transmission network to lower as much as possible the amount of load shed at bus 3. We remark that if generator 1 were to keep generating at its pre-contingency level (60 megawatts), then generator 2 would be capable of delivering 45 megawatts only, leading to a corresponding loss-of-load of 5 megawatt-hours at bus 3. This would occur because the transmission line flow limits (55 megawatts for each line) have to be enforced during all of the post-contingency states.

All three line outages occurring during any one of $\tau \in \{1, 2, 3\}$ would lead to load shedding in the amount of 5 megawatt-hours during that hour. This is so because of the low probabilities and the low load shedding impacts associated with these events. Fig. 2.1 illustrates an example of how the reserves are deployed after the failure of line 2. In this case, generator 1 backs down by 5 megawatts to comply with the line flow limits, while generator 2 stays off and generator 3 still outputs 50 megawatts.

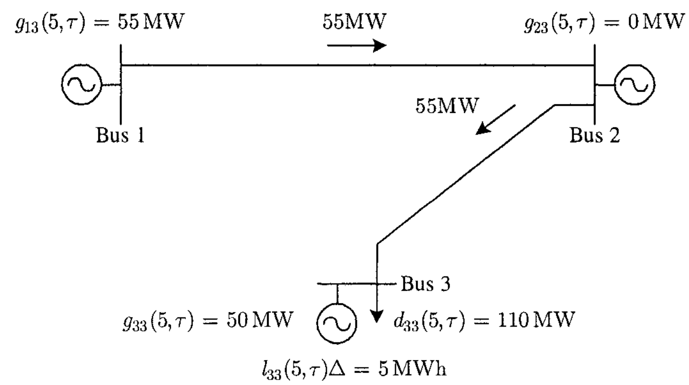

Fig. 2.1 Effect of the outage of line 2 ($k = 5$) during hour $t = 3$ if it occurs during $\tau \in \{1, 2, 3\}$

Hour $t = 4$

The loss of generator 3 during $\tau \in \{1, 2, 3, 4\}$ is taken care of by the 40 megawatts of non-spinning up reserve supplied by generator 1. The line outages occurring during any one of $\tau \in \{1, 2, 3, 4\}$ cannot lead to loss-of-load. Finally, the failure of generator 1 or 2 cannot cause loss-of-load either.

Stochastic versus deterministic schedules

Here we compare the results obtained with the stochastic approach to those of a purely deterministic security-constrained market-clearing scheme (as described in Section 2.2) for which load shedding is not allowed. Table 2.3 indicates that for the deterministic schedule generator 1 gets to provide the extra 30 megawatts of non-spinning up reserve required to cover the failure of generator 3 during the first hour. In addition, during hour $t = 3$, the demand at bus 3 provides 5 megawatts of up-going spinning (voluntary) reserve to compensate for any of the line losses. We notice also that these 5 megawatts are used in the aftermath of the outage of generator 1 or 3 during $\tau \in \{1, 2, 3\}$. This provides an explanation as to why generator 2 supplies only 55 megawatts of up-going non-spinning reserve during $t = 3$, rather than the 60 megawatts previously seen in Table 2.2 for the stochastic case. Thus, for example, the loss of generator 1 would be covered by 55 megawatts from generator 2 along with 5 megawatts from a voluntary reduction in demand at bus 3. We ought to remark, however, that the corresponding cost incurred by scheduling these 5 megawatts of demand-side reserve during $t = 3$ ($100) is much higher than the expected cost associated with load shedding of 5 megawatt-hours in the aftermath of a line failure ($4.40).

Table 2.3 Pre-contingency generation and reserves under deterministic market-clearing—System A

		Time t (h)			
		1	2	3	4
g_{1t}	(MW)	0.0	30.0	60.0	0.0
r_{1t}^{up}	(MW)	0.0	20.0	0.0	0.0
r_{1t}^{dn}	(MW)	0.0	0.0	5.0	0.0
$\tilde{r}_{1t}^{up}$	(MW)	30.0	0.0	0.0	40.0
$\tilde{r}_{1t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{2t}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{2t}^{up}$	(MW)	0.0	30.0	55.0	0.0
$\tilde{r}_{2t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{3t}	(MW)	30.0	50.0	50.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{up}$	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
d_{3t}	(MW)	30.0	80.0	110.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	5.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0

Next, Table 2.4 gives a comparison of the expected social cost breakdown of the stochastic schedule to that of the expected social cost of the deterministic schedule, which is given by the expression

$$p(0)W_{det}^{*} + \sum_{\tau=1}^{T} \sum_{k=1}^{K} p(k,\tau)W_{det}^{*}(k,\tau). \tag{2.22}$$

In (2.22), W^*_{det} represents the optimal social cost of the deterministic schedule and the terms $W^*_{det}(k,\tau)$ represent the reserve deployment social costs associated with the contingency scenarios being considered. The term $p(0)W^*_{det}$ refers to the expected cost incurred in the case when no contingencies occur, whereas the terms $p(k,\tau)W^*_{det}(k,\tau)$ correspond to the expected costs incurred when reserves need to be deployed under the contingency scenarios (k,τ). In the case of the deterministic schedule, we recall that the reserve deployment actions are not optimized during the scheduling process, unlike in the stochastic case.

Table 2.4 shows that the "Pre-contingency" and the "Reserve deployment" expected cost components of the stochastic schedule are lower than their counterparts associated with the deterministic schedule. Yet, the expected cost associated with load shedding in the stochastic schedule ends up diminishing these gains. The expected cost efficiency gain of the stochastic programming solution versus that of the deterministic solution, known as the value of the stochastic solution (VSS) [85], is equal to \$200.72 + \$2.00 − \$122.07 = \$80.65, representing 1.1% of the net deterministic schedule expected cost. This result demonstrates that when one takes into account the probabilities of failure, it is possible pre-position the system and deploy reserves at a lower expected cost, while still achieving a high level of reliability on average (as measured by the ELNS).

Table 2.4 Comparison of expected social costs under deterministic and stochastic market-clearing—System A

	Pre-contingency	Reserve deployment	Loss-of-load
Deterministic (\$)	7068.87	240.51	0.00
Stochastic (\$)	6868.15	238.51	122.07
Difference (\$)	200.72	2.00	−122.07

Prices of energy and security

Market-clearing procedures are never complete without the specification of a set of pricing rules such as locational marginal pricing for energy [3, 13], a pricing scheme that has gained industry acceptance and is now widely used. However, it is still unclear how reserve services should be remunerated and paid for [34, 58–64].

Recent works on security-constrained electricity market-clearing problems [34, 64] propose that *all* the reserve services supplied at a bus should be priced at the corresponding nodal marginal social cost of security. The central premise of this approach, under which there exists a single per-bus price for all reserve services, comes from the idea that in securing the energy supply at a bus, *all* the reserve services act in a concerted manner.

Here, we use the marginal pricing rules of [64], which are summarized in Appendix G and are thoroughly analyzed in Chapter 3 in the context of market-clearing with stochastic security. This proposition diverges from current practices in which the different reserve services—up/down and spinning/non-spinning reserves—are valued at different marginal rates [23, 28, 30, 58, 59, 63].

In the first three rows of Tables 2.5 and 2.6, we find the corresponding nodal prices of security and energy for the schedule based on market-clearing with stochastic security. We note some of nodal price differences during the third hour because of the transmission line congestion that follows the random failure of any one of the lines or that of generator 3.

Table 2.5 Prices of security—System A

		Time t (h)			
		1	2	3	4
		Stochastic schedule (\$/MWh)			
	1	8.11	6.01	1.27	12.89
Bus m	2	8.11	6.01	6.82	12.89
	3	8.11	6.01	7.23	12.89
		Deterministic schedule (\$/MWh)			
	1	12.60	6.01	1.71	12.90
Bus m	2	12.60	6.01	19.66	12.90
	3	12.60	6.01	19.35	12.90

We compare next the nodal prices obtained for the deterministic and stochastic market-clearing formulations. Before venturing any further, however, we remark that this comparison cannot be readily made since the sets of prices that correspond to the deterministic market-clearing would be derived from a mathematical programming problem with a different objective function. Therefore, we require to unbias the comparison by computing the corresponding deterministic prices of security and energy from the expected cost of the

Table 2.6 Prices of energy—System A

		Time t (h)			
		1	2	3	4
		Stochastic schedule (\$/MWh)			
	1	23.84	30.60	35.41	24.50
Bus m	2	23.84	30.60	40.96	24.50
	3	23.84	30.60	41.37	24.50
		Deterministic schedule (\$/MWh)			
	1	24.50	30.60	35.41	24.50
Bus m	2	24.50	30.60	53.37	24.50
	3	24.50	30.60	53.49	24.50

deterministic schedule, as calculated using (2.22), from small perturbations of the power balance relations of the deterministic market-clearing formulation. In other words, we obtain the nodal prices of energy for the deterministic schedule in Table 2.6 by calculating the ratios of the increments in the expected social cost of the deterministic schedule, caused by small demand perturbations affecting both the pre- and post-contingency power balance equality constraints (2.2) and (2.4), to those corresponding demand perturbations. We calculate the deterministic nodal prices in Table 2.5 using the same principle, but this time only perturbing the post-contingency power balance equality constraints (2.4).

If we compare the first three rows of Table 2.5 to its last three, we see that during hours 1 and 3, periods during which involuntary load shedding is used in the stochastic schedule, the prices of security associated with the stochastic schedule are much lower than those associated with the deterministic schedule. During hours 2 and 4, however, the prices of security are identical up to one cent per megawatt-hour. This illustrates that in some circumstances, given the value of lost load and the probabilities of the contingency scenarios, it is incrementally less expensive on average to shed load than to schedule and deploy the incrementally more expensive reserve services.

Performing the same comparison, but this time with the prices of energy, from Table 2.6 we observe once more that the stochastic prices are less than or equal to those of the deterministic schedule. As with the prices of security, we notice that during hours 1 and 3 the stochastic prices of energy are lower than those of the deterministic schedule. During period 3, for example, the price of energy corresponding to the stochastic schedule at bus 3 is 23% lower on average than the one corresponding to the deterministic schedule. These price differentials can be interpreted as premiums provided to the consumers for running the risk of being cut.

Impacts of key formulation parameters

Non-spinning reserve

Including non-spinning reserve services in the market-clearing formulation requires the explicit optimization of the on/off status of the generators for all contingency scenarios. This is a characteristic which, for large systems, may lead to computationally intractable problems, as was mentioned before in Section 2.4.2.

In order to evaluate the impact of ignoring non-spinning reserve in the current small system, we resolved the market-clearing problem again, but this time imposing that $u_{it}(k,\tau) = u_{it}$ for all $i \notin \mathcal{C}_k$, $k = 1, \ldots, K$ and $t \geq \tau$. The first two rows of Table 2.7 show how the expected social cost breaks down without non-spinning reserve. By comparing these results with the previous social cost decomposition with non-spinning reserve found in the following two rows, we observe that most of the expected social cost components have increased. The expected social cost component that corresponds to the post-contingency reserve deployment, however, has gone down by 4.35% because now this component no longer accounts for post-contingency generator startup costs. It is interesting to note as well that the expected social cost related to involuntary load shedding has increased significantly (182% of the original expected social cost with non-spinning reserve). This acute jump can be explained by inspecting how the reserves and generation schedules, shown in Table 2.8, have been modified from the original case with spinning reserve (reported in Table 2.2). It is seen that the failure of generator 1 is no longer covered during hour 2; thus, this causes a 30-megawatt-hour deficit for that period, unlike previously when generator 2 was providing the 30 megawatts of non-spinning reserve to cover that contingency.

This behavior is a clear example of the balancing act operating between security and economics. Indeed, one can show that only if the value of lost load is raised up and above \$1247 per megawatt-hour then the ELNS can be brought back to its original level found in the case with both reserve types. Nevertheless, this improvement in security is costly, as it leads to an increase in the expected cost of pre-contingency operation to \$7395.35 from

$6868.15, and from $238.51 to $241.27 for post-contingency reserve deployment.

Table 2.7 Breakdown of expected social costs when excluding non-spinning reserve—System A

	Total	Pre-contingency	Reserve deployment	Loss-of-load
	Excluding non-spinning reserve			
Cost ($)	7721.07	7148.68	228.13	344.26
% total cost	100.00	92.59	2.95	4.46
	With non-spinning reserve			
Cost ($)	7228.74	6868.15	238.51	122.07
% total cost	100.00	95.01	3.30	1.69
Difference ($)	492.33	280.53	−10.38	222.19
Difference (%)	6.38	4.08	−4.35	182.00

Value of lost load

We just saw that by raising the value of lost load (VOLL), it is possible to improve the security of the supply as measured by the ELNS, which generally decreases. Of course, usually this has some cost. Fig. 2.2 illustrates quite well how the ELNS and the expected social cost vary with the VOLL. Initially, for low values of the VOLL, the ELNS is somewhat insensitive to changes in the VOLL, only with a slight tendency to decrease monotonically. Then, in the range between $300 and $1000 per megawatt-hour, the ELNS starts to decrease sharply as, at the same time, the corresponding expected social cost of load shedding begins to represent a more significant portion of the total expected social cost in comparison to those of reserve scheduling and deployment.

In cases when isolated load pockets are created during post-contingency states, raising the VOLL in those load pockets is not sufficient to improve security in the ELNS sense. The only recourses available to improve the security of supply in those load pockets involve longer-term measures like the addition of transmission lines and local generating stations, as well as the implementation of more elaborate and aggressive demand-side management programs.

Table 2.8 Pre-contingency generation, reserves and ELNS when excluding non-spinning reserve—System A

		Time t (h)			
		1	2	3	4
g_{1t}	(MW)	0.0	30.0	50.0	10.0
r_{1t}^{up}	(MW)	0.0	50.0	10.0	30.0
r_{1t}^{dn}	(MW)	0.0	0.0	5.0	0.0
g_{2t}	(MW)	0.0	0.0	10.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	45.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
g_{3t}	(MW)	30.0	50.0	50.0	30.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
d_{3t}	(MW)	30.0	80.0	110.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_{3t}$	(kWh)	117.7	116.8	32.1	77.7

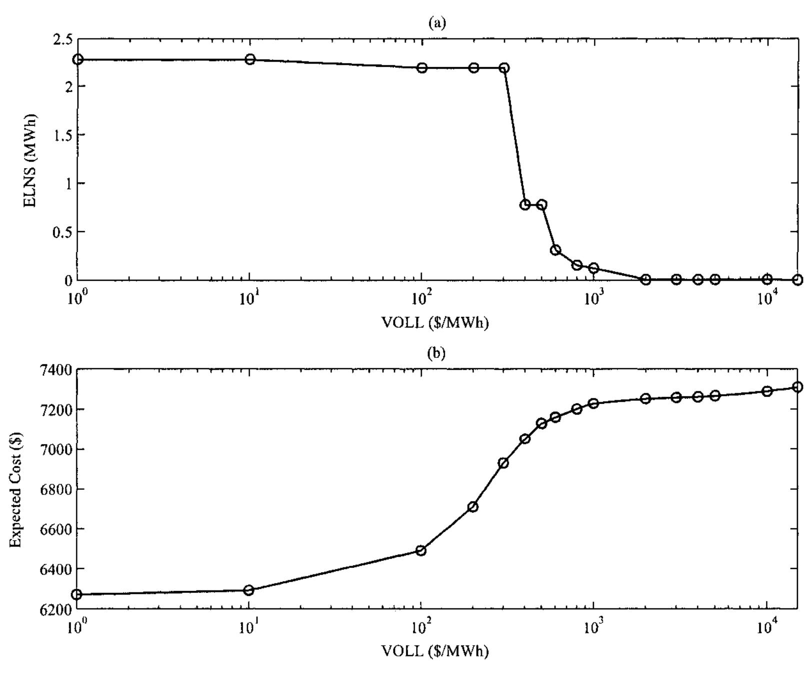

Fig. 2.2 Effect of the magnitude of the value of lost load at bus 3: (a) on the ELNS; (b) on the total expected social cost

Ramping limits

Ramping limits are of prime concern in time-dynamic generation scheduling problems, especially those that jointly schedule energy and reserve services. In the previous test cases, the generators were assumed not to be limited by ramping, allowing them to ramp from zero to their maximum capacity, $g_i^{\max}$, and back to zero within a single time period.

Here, let us assume that generator 1 now has a ramping limit of 29 megawatts per hour, with generators 2 and 3 still having the possibility to change their output at the rates of 100 and 50 megawatts per hour respectively. We further let these ramping rates apply for both up- and down-going ramping, including startup and shutdown.

Comparing the schedule in Table 2.9 to the original one—with relaxed ramping limits, as shown in Table 2.2—, we observe major modifications in the schedule. The most noticeable difference is the requirement that generator 2 be brought online during hour $t = 3$. In addition, the ramp limitations of generator 1 restrict it from turning off during period $t = 4$. This results into the need for generator 3 to back off by 21 megawatts because generator 1 is restricted from ramping further down. Furthermore, we notice the scheduling of 2.5 megawatts of demand-side up-spinning reserve during the third hour. The purpose for scheduling this reserve, which costs \$20 per megawatt-hour, is to counter the loss of a line. If we look more closely, it serves the extra purpose of avoiding the deployment of reserve from generator 2, whose deployment cost is quite high at \$40 per megawatt-hour.

Set of pre-selected contingencies

The membership of the pre-selected set of contingencies can potentially have important impacts on the optimal market-clearing outcome. For example, the addition a new contingency to an already existing set decreases the probability that no contingency occurs, while at the same time it increases the probability that at least one contingency occurs. Thus, this leads the pre-contingency expected social cost term, proportional to $p(0)$, to assume less importance in the market-clearing objective function (2.11). On the other hand, the reserve deployment term ends up carrying more weight, as the objective function then accounts for the extra expected cost associated with the probable occurrence of the new contingency. Also, the new contingency may require additional involuntary load shedding if its corresponding expected cost increment is less than the expected costs of scheduling more reserve plus those associated with its deployment. We remark, however, that by

Table 2.9 Pre-contingency generation, reserves and ELNS under ramp limits—System A

		Time t (h)			
		1	2	3	4
g_{1t}	(MW)	10.0	30.0	50.0	21.0
r_{1t}^{up}	(MW)	0.0	9.0	0.0	0.0
r_{1t}^{dn}	(MW)	0.0	0.0	5.0	0.0
$\tilde{r}_{1t}^{up}$	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{1t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{2t}	(MW)	0.0	0.0	10.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	47.5	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{2t}^{up}$	(MW)	10.0	41.0	0.0	21.0
$\tilde{r}_{2t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
g_{3t}	(MW)	20.0	50.0	50.0	19.0
r_{3t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{up}$	(MW)	0.0	0.0	0.0	0.0
$\tilde{r}_{3t}^{dn}$	(MW)	0.0	0.0	0.0	0.0
d_{3t}	(MW)	30.0	80.0	110.0	40.0
r_{3t}^{up}	(MW)	0.0	0.0	2.5	0.0
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_{3t}$	(kWh)	39.2	0.0	45.2	0.0

adding one more contingency to an existing pre-selected set, one cannot make worse the value of the *true* ELNS that considers all possible contingency scenarios.

As an illustration of the above assertions, consider adding all double generator outages (assumed to occur independently and simultaneously) to the original set of pre-selected contingencies (comprising all single generator and line failures). In this case, the optimal pre-contingency generation and reserve schedule found by stochastic market-clearing is identical to the one already obtained with the original set of contingencies reported in Table 2.2. What is striking here is that even though there are several more contingency scenarios, no extra reserve services were scheduled. However, as a result of that, here all the hourly levels of ELNS have gone up as shown in Table 2.10 (for a total of 128.4 kilowatt-hours representing again about 0.05% of the total scheduled energy consumption over the entire horizon).

The small increases in ELNS associated with the extra contingencies did not justify scheduling and deploying any more reserves. This shows that, when one uses stochastic market-clearing, new contingencies may not be fully covered in the deterministic sense (indeed here, market-clearing based on a deterministic security criterion is infeasible). When considering extra contingencies under stochastic market-clearing, additional reserve services are scheduled only if their corresponding expected costs of scheduling and, particularly, of deployment are less than those of load shedding.

Finally, Table 2.11 compares the expected social cost breakdown with the extra contingencies to the cost breakdown corresponding to the schedule found with the original set of contingencies. As conjectured above, it demonstrates that the proportions of the expected social cost components have shifted from the pre-contingency toward the post-contingency control actions.

Table 2.10 ELNS under an alternate set of contingencies—System A

	Time t (h)			
	1	2	3	4
	Alternate set of contingencies			
$\hat{l}_{3t}$ (kWh)	118.2	1.5	7.4	1.3
	Original set of contingencies			
$\hat{l}_{3t}$ (kWh)	117.7	0.0	4.4	0.0

Table 2.11 Breakdown of expected social costs under an alternate set of contingencies—System A

	Total	Pre-contingency	Reserve deployment	Loss-of-load
	Alternate set of contingencies			
Cost ($)	6913.54	6546.27	238.92	128.36
% total cost	100.00	94.69	3.46	1.86
	Original set of contingencies			
Cost ($)	7228.74	6868.15	238.51	122.07
% total cost	100.00	95.01	3.30	1.69

2.5.2 IEEE RTS study

In this section, electricity market-clearing with stochastic security is tested over a 24-hour scheduling horizon on the IEEE RTS [67] (System B), described in Appendix E.2.

Here, the set of credible contingencies is made up of all the single failures of generators having a capacity greater than or equal to 197 megawatts: the three U197 units located at bus 13, the U350 unit located at bus 23 and the two U400 units located at buses 18 and 21 respectively. This limited pre-selected set is justified because it contains some of the contingencies having the most severe impact on the system-wide expected load not served [52]. Like in the previous small-scale case study, here we assume that the involuntary interruption duration parameter Δ is one hour-long.

Computational complexity

As for the small-scale study reported in Section 2.5.1, this example was solved using the MILP solver CPLEX (version 9.0.2) running under GAMS. Given a pre-specified solution tolerance gap of 1%, the CPU solution time was 40 minutes and 41 seconds on *Ampère*. See Appendix F for descriptions and relevant references about GAMS, CPLEX and *Ampère*.

The dimensions of the problem being studied are remarkable: a total of 1 018 033 variables—576 of which are binary[4]—and 1 754 981 constraints. Table 2.12 reports on the breakdown of the main contributors to the formulation's variable and constraint counts. We should point out that the numbers found in Table 2.12 are those calculated before the pre-processing engine of CPLEX is run. We recall that the role of the solver's pre-processing engine is to eliminate redundant constraints (rows) and optimization variables (columns) with the ultimate goal of reducing the computing hardware's core memory allocation requirements and the CPU time. In fact, after pre-processing, the array size of the market-clearing problem is reduced to the more manageable size of 383 276 rows and 297 079 columns. Some instances of redundant variables include the fixed voltage angle of the reference bus or the pre-contingency load shedding variables set equal to zero in (2.16). Examples of redundant constraints include ramping limitations that apply to generators forced to remain off because of a minimum down-time restriction.

Table 2.12 Breakdown of the dominant post-contingency variables and constraints—System B

Variable	Count
Voltage angles	43 200
Load shedding	43 200
Net power output	340 224
Power blocks	428 544
Constraint	**Count**
Power balance	43 200
Sum over the power blocks	107 136
Power flow limits	273 600
Upper bounds on power blocks	428 544

From Table 2.12, it is obvious that, in spite of the limited size of the IEEE RTS, the dimensions of the corresponding stochastic market-clearing problem are important. We note, however, that some reductions can be made to palliate the associated dimensional explosion. One such mitigation measure would use a reduced zonal description of the grid.

[4]The number of binary variables reflects the fact that eight of the generators (six U50 and two U400) were assumed to be must-run, *i.e.* $u_{it} = 1$ for $t = 1, \ldots, T$. Moreover, as shown in Appendix B, the unit commitment formulation we used did not require any extra binary variables to model the startup and shutdown of generating units. Thus, since the problem spreads over a 24-hour horizon and schedules $32 - 8 = 24$ generating units, then the problem formulation contains a total of 576 binary variables.

Likewise, decreasing the number of pre-selected contingencies can reduce the dimensions of the problem, under the assumption that it is always better to consider a few significant contingencies than none at all. Furthermore, as mentioned before in Section 2.4.2, scenario reduction techniques [105, 121–123] are certainly promising, as well as is the large family of decomposition techniques [27, 31, 82, 85, 112, 115–120]. Lastly, one cannot neglect the constant improvements in the cost and performance of computing machinery and mixed-integer optimization codes [124]. These advances allowed the development of the current large ISOs in the United States of America and Canada (PJM, New York ISO, ISO New England and Ontario being the prime examples)—something, of course, that was not conceivable even twenty years ago.

Results and analysis

As it was done before with System A, Table 2.13 compares the breakdown of the expected social cost of the optimal stochastic schedule to that of the deterministic schedule for which involuntary load shedding is not permitted. As before, the expected cost of the deterministic schedule is computed using (2.22). By inspection of Table 2.13, we note that for the stochastic schedule the "Pre-contingency" and "Reserve deployment" expected costs, particularly the latter, are lower than the corresponding expected costs under the deterministic market-clearing. These gains, however, are reduced by \$161, corresponding to the expected cost of involuntary load shedding. Nevertheless, when combining all three cost components, we determine that the stochastic solution provides a net expected economic improvement, or value of the stochastic solution (VSS), of \$4301 (representing about 1% of the expected social cost under deterministic market-clearing). This example shows very well the economic advantage brought about by the co-optimization of post-contingency actions, since here the expected savings represent a good 7% (\$4304 − \$161 = \$4143) of the expected reserve deployment costs under the deterministic schedule (\$56 602).

In addition, we point out that the only instances of involuntary load shedding in the stochastic schedule are applied at buses 8 and 20 (in the expected amount of 40.4 kilowatt-hours at each bus) during hour 1, an off-peak hour during which the value of lost load is the lowest (\$2000 per megawatt-hour versus \$3000 per megawatt-hour during the on-peak hours).

Table 2.13 Comparison of expected social costs under deterministic and stochastic market-clearing—System B

	Pre-contingency	Reserve deployment	Loss-of-load
Deterministic (\$)	379 540	56 602	0
Stochastic (\$)	379 382	52 298	161
Difference (\$)	158	4 304	−161

Prices of energy and security

Fig. 2.3 (a) illustrates how the marginal prices of energy and security, for all buses except bus 7, vary with time, while Fig. 2.3 (b) demonstrates the price evolution at bus 7. We observe a major price spike happening during hour 9 as security prices climb up to \$43.46 per megawatt-hour and energy prices to \$51.29 per megawatt-hour at all buses except at bus 7, where the prices remain low at \$5.84 per megawatt-hour for security and \$13.67 per megawatt-hour for energy. The conjunction of a number of factors explains this price spike. First, during hour 9, the three U100 generators located at bus 7 (see Fig. E.2) have plenty of inexpensive up-spinning reserve available. Nevertheless, this reserve capacity cannot be fully deployed because of the flow limit of 175 megawatts on the transmission line tying this bus to the rest of the power system. Second, also during that hour, two out of the three incrementally cheap U197 generators (all located at bus 13) have to be maintained offline because of minimum down-time constraints. The end result is that with so little reserve capacity available, the system operator has to (i) bring online the four very expensive U20 generators; and, (ii) schedule 1.71 megawatts of expensive demand-side up-spinning reserve at bus 13.

2.6 Summary

In this chapter, we proposed and formulated an electricity market-clearing scheme with multi-period unit commitment that integrates a stochastic security criterion. The stochastic security criterion is based on the probabilities of occurrence of pre-selected sets of generator and line outages with known historical failure rates as well as for random demand disturbances. Unlike in the deterministic security-constrained market-clearing problem with unit

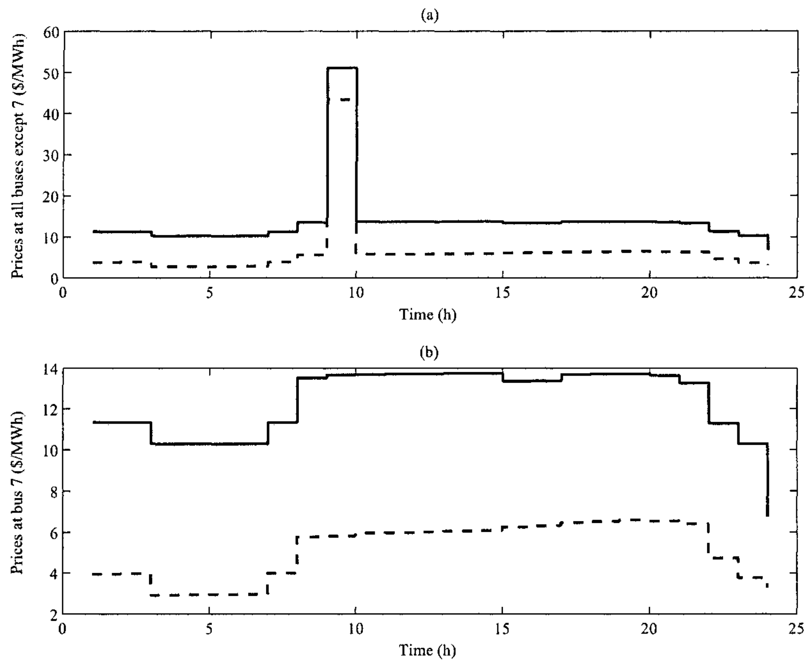

Fig. 2.3 Marginal prices of energy (solid line) and security (dashed line): (a) at all buses except bus 7; (b) at bus 7.

commitment, also described in this chapter, here the required levels of reserve services are determined by economically penalizing the operation of the market by the expected social cost associated with post-contingency involuntary load shedding events.

This formulation is a stochastic programming problem that contains several significant generalizations over earlier work in the field of security-constrained market-clearing, namely: (i) the consideration of time-dynamic generation constraints like minimum up- and down-time as well as ramping limits; (ii) a dc network model to account for pre- and post-contingency transmission line flow limits; (iii) the explicit consideration of involuntary load shedding as well as voluntary consumption adjustments in the form of demand-side reserve; (iv) the addition of the time dimension to contingencies as they may occur randomly within the scheduling horizon; (v) a stochastic market-clearing objective function that measures the expected pre-contingency social cost added to the expected post-contingency social cost associated with the corrective rescheduling and involuntary load shedding actions; and, (vi) a security index defined by the expected load not served due to random line and generator outages calculated without the need to define any extra binary variables on top of the classic generator on/off binary variables.

The underlying philosophy here acknowledges the complex engineering and economical couplings that render the operation of large-scale power systems a challenging task. In addition, the proposed market-clearing scheme recognizes that the management of randomness in power system operation constitutes one of the core elements of this complicated task. Therefore, power system operation planning based on a stochastic security criterion is advantageous because it provides system operators with gauges about the likelihood and expected consequences of contingencies as well as response plans for post-contingency actions and their corresponding expected costs. The results of the two case studies have illustrated these advantages.

In the first case study, we investigated how a number of factors impact on the generation and reserves schedules found via market-clearing with stochastic security. These factors included: (i) transmission line flow limits; (ii) the exclusion of non-spinning reserve; (iii) the demand-side value of involuntary load shedding; (iv) generator ramping limits; and, (v) the composition of the pre-selected set of contingencies. To demonstrate the superiority of market-clearing with stochastic security over its deterministic counterpart, we compared their respective scheduling results and expected costs. One important finding is that the expected social costs of preventive and corrective security control actions are lower in the stochastic case even though these efficiency gains are slightly eroded by the expected costs of involuntary load shedding actions.

Likewise, we saw that the balancing act between reliability and social cost minimization depends strongly on the consumers' valuation of involuntary load interruptions. Another worthy observation is that the expected prices of energy and of security derived from the stochastic schedule are lower than those derived from the deterministic schedules, under which load shedding is not permitted. This is fundamental in the sense that as there is a risk of involuntary load shedding, the consumers get to pay less for security and energy.

In the second case study, we ran a 24-hour scheduling of the IEEE Reliability Test System based on market-clearing with stochastic security. This study illustrated that, in large-scale market-clearing problem instances, the system operator should try to restrict the number of pre-selected contingencies and their associated time of occurrence within the scheduling horizon. An inspection of the solution results provided by the commercial mixed-integer linear solver used to resolve the market-clearing problems showed that often many of the constraints associated with particular contingencies remain inactive. This is a property that warrants further investigation of so-called umbrella contingencies, which we attempt later in Chapter 5. In addition, solution techniques based on decomposition methods should be promising strategies because they do not require that the entire problem array be located in core memory, and they allow for the exploitation subproblem parallelism. Lastly, we suggested that scenario reduction techniques for stochastic optimization problems, applied specifically to electricity market-clearing with stochastic security, be investigated.

Chapter 3

Pricing of Energy and Security Under Market-Clearing With Stochastic Security

> The real price of every thing, what every thing really costs to the man who wants to acquire it, is the toil and trouble of acquiring it. What every thing is really worth to the man who has acquired it, and who wants to dispose of it or exchange it for something else, is the toil and trouble which it can save to himself, and which it can impose upon other people.
>
> *Wealth of Nations*
> *Adam Smith, 1723–1790*

3.1 Introduction

The design of a market-based electricity scheduling scheme is never complete without the specification of a set of pricing rules. Over the last decade, as the electricity industry restructured, a number of electricity pricing schemes have been proposed and implemented among which we distinguish two fundamental philosophies: one based on *marginal pricing* and the other based on *pay-as-bid pricing*.

3.1.1 Marginal pricing

The foundations of marginal pricing are strong in the field of theoretical microeconomics [125]. For a given market, under somewhat restrictive assumptions of convexity of the feasible production sets of suppliers and of concavity of the utility functions of consumers, it can be shown that there exists a single equilibrium price, equal to the marginal cost of production or, equally, the marginal consumer utility, achieved when supply and demand meet. It is indeed easy to show that, under these market equilibrium conditions, the Marshallian aggregate surplus (*i.e.* social welfare) is maximized, while the market allocation associated with the single uniform marginal price is Pareto optimal. This result is often called the *First Fundamental Theorem of Welfare Economics* [125].

More fundamentally yet, the concept of marginal pricing parallels the notion of duality in constrained optimization theory [82, 126]. For a given constrained optimization problem, one can define, associated with each constraint, a so-called dual variable or Lagrange multiplier. At the optimum of the mathematical programming problem, the values of the associated dual variables represent the marginal change in the value of the objective function brought about by small perturbations of their corresponding right-hand sides. In the case of an electricity market-clearing problem, the Lagrange multiplier associated with a bus power balance constraint is equal to the marginal change in the optimum social cost due to a corresponding marginal change in the power balance relation at that bus. In other words, this Lagrange multiplier represents the marginal value of balancing power at that bus, which, under marginal pricing, becomes the price of electricity [3].

Under marginal pricing of electricity, each generator at a given bus earns a revenue equal to its generation level times the prevailing bus marginal price, while consumers pay an amount equal to their consumption times the price of electricity at that bus. Thus, generators have a profit margin from the difference between the bus price and their marginal cost of production, while correspondingly the loads obtain a benefit margin from the difference between their marginal utility and the bus price. As mentioned above, however, the existence of marginal prices that allow consumer utility and generator profit maximization, while balancing supply and demand, is only guaranteed under restrictive convexity conditions of the production technologies.[1] As shown in [127–129], this condition is not satisfied in general for electricity generation technologies. As a result, for given marginal prices, generators may have profit-based incentives to deviate from their schedule found by the centralized market-clearing problem. If these deviations were to materialize, the security of the power system could be challenged as supply may not equal demand anymore. This is why remuneration by marginal pricing often has to be complemented with some out-of-market monetary transfers (generally operated from consumers to generators) known as uplifts [127–129]. This issue lies outside of the current dissertation; we refer the interested reader to [127–129].

3.1.2 Pay-as-bid pricing

Unlike marginal pricing, the pay-as-bid pricing regime avoids the need for any kind of out-of-market monetary transfers in the form of uplifts. Under pay-as-bid pricing, generators at a bus are not remunerated at a uniform rate, rather they are compensated according to their cost-based bids.[2] This remuneration scheme thus pre-supposes that the generator offers are already marked up to assure a given level of profit, which is independent of the marginal value of electricity.

One criticism of pay-as-bid pricing is that it may give perverse incentives to generators to greatly overstate their generation cost offers with the objective of reaping higher profits and consequently hiking the corresponding consumer payments. However, several authors, for example Ren and Galiana [130, 131], showed that pay-as-bid pricing does not necessarily—at least in theory—lead to significantly higher consumer payments. In particular, these authors demonstrated that as the number of generators gets large, the consumers end up paying as much as under marginal pricing. In addition, these authors also showed that pay-as-bid pricing can potentially reduce the volatility of the consumer payments.

Notwithstanding these apparent advantages, pay-as-bid pricing has not received widespread industry interest so far. To this day, the only practical implementation of pay-as-bid pricing has been in the United Kingdom, where, under the New Electricity Trading Agreements, it is used to remunerate generators in the daily balancing market [132]. At the time of writing, energy regulatory authorities in the Islamic Republic of Iran are contemplating the use of pay-as-bid pricing for their developing electricity market [133].

[1] The concavity assumption of demand-side utility functions is usually satisfied.

[2] The more semantically-correct term here would be "offers".

3.1.3 Locational marginal pricing

Nowadays in North America, all FERC-approved ISOs have to value electricity based on locational (nodal) marginal prices [3], obtained as byproducts of network-constrained market-clearing. Network-constrained market-clearing and locational marginal pricing, unlike single-node or zonal-based pricing models, recognize the complicating aspects that the transmission network introduces in the market-based trading of electricity. These complications include (i) most commonly line congestion problems [9, 11, 13, 68, 134, 135]; and, (ii) possibly transmission losses [9, 11, 68]. Generally, two main criticisms stem out of the explicit consideration of the transmission grid as part of the market-clearing solution and pricing processes. The first criticism is the obvious increase in the computational burden of the market-clearing and pricing procedures, while the second concerns the dubious transparency of these procedures in light of the augmented complexity. Nevertheless, studies have shown that the improvements in economic efficiency brought about by the more complex network-constrained market-clearing and locational marginal pricing can be quite significant, a result that contradicts the arguments in favor of the simpler market models [110, 136].

3.1.4 On pricing of security

Recently, in response to the blackouts that affected several large grids in the Western world [15–19], good parts of the industry, government regulators and researchers have re-shifted their attention to the security aspects of power system operation. This change in focus had to be made at the expense of the marketing and profit motives that had concentrated attention since the beginnings of the industry restructuring efforts. Nevertheless, the current shift in attention occurs with the basic assumption that market-based operation of power systems is here to stay and that any proposed innovations should attempt to better reconcile the market and the secure operation objectives. The proposed methods in this dissertation are certainly following that philosophy.

Now that the focus of the power system operation community has shifted from *network-constrained* to *security-constrained* market-clearing, corresponding shifts must be operated with respect to the pricing aspects of security. The current industry practice in North America assigns different prices to a number of "security-enhancing" ancillary services [10, 23, 24, 26–28, 30, 31, 35, 58–60, 63], which include the different reserve services described in previous chapters and the supply of reactive power, for example.

This pricing philosophy is somewhat flawed as it fails to recognize that power system security is maintained through the *coordinated* scheduling and deployment of *all* these security-enhancing ancillary services. Therefore, one can argue in favor of valuing the ancillary services at the *marginal price of security.* In order to do so, it is required to define rigorously the concept of power system security. We adopt the definition put forward by Arroyo and Galiana in [64], which is elaborated further with Bouffard and Restrepo in [34].

Definition 3.1 (Power system security). *Power system security* is the ability of the system to balance power at every bus while satisfying all operational limits associated with given regulation intervals (primary, secondary and tertiary)[3] in the aftermath of any credible contingency.

A direct consequence of this definition is that there should be no distinction between the means taken to enforce power system security. In fact, instead of pre-specifying some levels of reserves following some rule-of-thumb, it is the set of pre- and post-contingency power balances that determines the requirements for the various ancillary services, thus rendering the power system operator indifferent with respect to the means used to meet the security objective. Hence, constraints in the market-clearing formulation requiring minimal amounts of the various ancillary services are no longer needed, as seen in Chapter 2; this means that one cannot calculate individual marginal prices for the ancillary services from these constraints.

Therefore, under this proposal, the ancillary services are to be valued at the bus marginal prices of security, for which the necessary theoretical framework behind their computation is found in [34, 64]. For quick reference, the basis of this proposal is summarized in Appendix G.

Furthermore, under security-constrained market-clearing, the corresponding nodal marginal prices of energy become tightly bound to the bus marginal prices of security. Arroyo and Galiana demonstrated that the price of energy embeds that of security [64]. This fact makes perfect sense, as under security-constrained market-clearing, one does not simply buy a mere megawatt-hour of energy, but buys a *secure* megawatt-hour of energy.

In this chapter, we specialize the propositions and results of [64] to pricing of energy and

[3] In this dissertation, we only concern ourselves with the tertiary regulation interval. We refer the interested reader to [34, 36] for the treatment of the other regulation intervals.

security in the context of electricity market-clearing with stochastic security. We derive a number of results outlining the couplings existing between the value of lost load, involuntary load shedding actions and the probabilities of occurrence of contingencies. These results are of importance because they provide essential information as to why or why not involuntary load shedding is to be used after some contingencies. We discuss results for general nonlinear and linear market-clearing models, and we look at a simple numerical example illustrating the theoretical results just derived.

3.2 Marginal Pricing Under Market-Clearing With Stochastic Security

The market-clearing problem formulation with a stochastic security criterion, as the one developed in this dissertation, is a specific instance of the general security-constrained market-clearing problem found in (G.1)–(G.4) of Appendix G. In what follows, we examine the relationships existing between the various Lagrange multipliers associated with the constraints of the market-clearing problem, the presence of load shedding, the value of lost load as well as the probabilities of the contingencies. Without loss of generality here, we will only refer to the concept of contingency without explicitly differentiating between contingencies in the classical sense (as in Chapter 2) and contingencies in the sense of forecasting errors (as in Chapter 4). Moreover, to simplify the exposition here, we do not explicitly consider the time dimension. We note, however, that the results apply integrally with the time dimension added.

First, let us partition the vector of continuous decision variables $\mathbf{x}$

$$\mathbf{x} = [\mathbf{x}(0) \mid \mathbf{x}(1) \cdots \mathbf{x}(K) \mid \mathbf{l}(1) \cdots \mathbf{l}(K)]^T , \tag{3.1}$$

where: (i) $\mathbf{x}(0)$ is the sub-vector of generation, demand and network variables associated with the pre-contingency state; (ii) $\mathbf{x}(1), \ldots, \mathbf{x}(K)$ are the sub-vectors of generation, demand and network variables associated with the operation of the system under the preselected contingency scenarios; and, (iii) $\mathbf{l}(1), \ldots, \mathbf{l}(K)$ are the sub-vectors of involuntary load shedding also associated with the operation under the contingencies. A similar partition of the discrete variable vector is also defined

$$\mathbf{u} = [\mathbf{u}(0) \mid \mathbf{u}(1) \cdots \mathbf{u}(K)]^T . \tag{3.2}$$

Here, the objective function is a linear combination of the expected social costs associated with (i) the operation in the pre-contingency state; (ii) the operation following the occurrence of the credible contingencies; and, (iii) the post-contingency involuntary load shedding. We point out that each of the expected social cost components depends only on variables associated with its corresponding state. This permits re-expressing the generic objective (G.1) in the decomposed form

$$W(\mathbf{u}, \mathbf{x}) = p(0)W(\mathbf{u}(0), \mathbf{x}(0), 0) + \sum_{k=1}^{K} p(k)[W(\mathbf{u}(k), \mathbf{x}(k), k) + \mathbf{v}^T \mathbf{l}(k)], \tag{3.3}$$

where $p(0)W(\mathbf{u}(0), \mathbf{x}(0), 0)$ is the expected social cost component under the non-contingent state, $p(k)W(\mathbf{u}(k), \mathbf{x}(k), k)$ is the expected social cost of operation under contingency scenario k, and $\mathbf{v}$ is the vector of the values of lost load. Likewise, we recall that the power balance conditions of the generic problem, (G.2) and (G.3), depend only on the variables associated with their respective contingency scenarios

$$\mathbf{H}(\mathbf{u}(0), \mathbf{x}(0), \mathbf{0}, 0) = \mathbf{0}, \tag{3.4}$$

$$\mathbf{H}(\mathbf{u}(k), \mathbf{x}(k), \mathbf{l}(k), k) = \mathbf{0}; \quad k = 1, \ldots, K. \tag{3.5}$$

For notational compactness, we will use the notation $\mathbf{H}(k) \equiv \mathbf{H}(\mathbf{u}(k), \mathbf{x}(k), \mathbf{l}(k), k)$ and $W(k) \equiv W(\mathbf{u}(k), \mathbf{x}(k), k)$ for $k = 0, 1, \ldots, K$, as well as $\mathbf{G} \equiv \mathbf{G}(\mathbf{u}, \mathbf{x})$.

3.2.1 Fundamental results

Load shedding and Lagrange multipliers

Proposition 3.1 (Relations between load shedding and Lagrange multipliers). *Load shedding is applied at bus m after the occurrence of contingency k if and only if the Lagrange multiplier associated with the bus power balance under that contingency ($\mu_m(k)$) is greater than or equal to the quantity $p(k)v_m$.*

Proof. The optimal solution of the market-clearing $(\mathbf{u}^*, \mathbf{x}^*)$ must satisfy the first-order

Karush-Kuhn-Tucker (KKT) necessary optimality condition [126] with respect to all the continuous variable sub-vectors. In order to perform that assessment, consider the Lagrangian function of the mathematical program (G.1)–(G.4), with its objective given by (3.3), evaluated for the optimal value of the discrete variables, $\mathbf{u}^*$:

$$\begin{aligned}\mathcal{L}(\mathbf{u}^*, \mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\sigma}) = p(0)W(0) + \sum_{k=1}^{K} p(k)[W(k) + \mathbf{v}^T\mathbf{l}(k)] \\ - \boldsymbol{\mu}^T(0)\mathbf{H}(0) - \sum_{k=1}^{K} \boldsymbol{\mu}^T(k)\mathbf{H}(k) - \boldsymbol{\sigma}^T\mathbf{G}, \quad (3.6)\end{aligned}$$

where $\boldsymbol{\mu} = [\boldsymbol{\mu}(0)\ \boldsymbol{\mu}(1) \cdots \boldsymbol{\mu}(K)]^T$. For the load shedding variables $\mathbf{l}(k)$ associated with the $k = 1, \ldots, K$ contingencies, the corresponding first-order KKT conditions are

$$\frac{\partial \mathcal{L}}{\partial \mathbf{l}(k)} = p(k)\mathbf{v} - \left(\frac{\partial \mathbf{H}(k)}{\partial \mathbf{l}(k)}\right)^T \boldsymbol{\mu}(k) - \left(\frac{\partial \mathbf{G}}{\partial \mathbf{l}(k)}\right)^T \boldsymbol{\sigma} = \mathbf{0}. \quad (3.7)$$

In the post-contingency power balance relations (3.5), the load shedding sub-vectors appear linearly [as seen in (2.8)]. Thus, for each of the contingencies $k = 1, \ldots, K$, we can write

$$\mathbf{H}(k) = \mathbf{H}'(k) + \mathbf{l}(k) = \mathbf{0}, \quad (3.8)$$

where we assume that $\mathbf{H}'(k)$ does not depend on any of the load shedding variables. This implies that for $k = 1, \ldots, K$

$$\frac{\partial \mathbf{H}(k)}{\partial \mathbf{l}(k)} = \mathbf{I}, \quad (3.9)$$

where $\mathbf{I}$ is an $M \times M$ identity matrix. Moreover, we recall from (2.9) that the only bounds on the load shedding variables are (assuming that there are no upper bounds imposed on the ELNS)

$$\mathbf{0} \le \mathbf{l}(k) \le \mathbf{d}(k); \quad k = 1, \ldots, K. \quad (3.10)$$

As a result, we have for $k = 1, \ldots, K$

$$\frac{\partial \mathbf{G}}{\partial \mathbf{l}(k)} = \begin{cases} \mathbf{I} & \text{for the lower bound on } \mathbf{l}(k), \\ -\mathbf{I} & \text{for the upper bound on } \mathbf{l}(k), \\ \mathbf{0} & \text{otherwise.} \end{cases} \tag{3.11}$$

Thus, with the aid of (3.9) and (3.11), we can rewrite (3.7) as

$$\boldsymbol{\mu}(k) = p(k)\mathbf{v} - \boldsymbol{\sigma}^{lb}(k) + \boldsymbol{\sigma}^{ub}(k), \tag{3.12}$$

where $\boldsymbol{\sigma}^{lb}(k)$ and $\boldsymbol{\sigma}^{ub}(k)$ correspond respectively to the Lagrange multiplier vectors associated with the lower bounds and the upper bounds on $\mathbf{l}(k)$.

In addition to the first-order KKT condition (3.7), the optimal solution to the market-clearing problem and the Lagrange multipliers associated with the inequality constraints must satisfy the complementary slackness condition

$$\boldsymbol{\sigma}^T \mathbf{G} = 0, \tag{3.13}$$

and the non-negativity of the multipliers associated with the inequality constraints

$$\boldsymbol{\sigma} \geq \mathbf{0}. \tag{3.14}$$

Now, we consider the following two cases:

1. There is load shed at some bus m due to contingency k, that is $l_m(k) > 0$. Complementary slackness (3.13) and the sign restriction (3.14) therefore require that

$$\sigma_m^{lb}(k) = 0, \tag{3.15}$$

$$\sigma_m^{ub}(k) \geq 0. \tag{3.16}$$

Thus, under these conditions, (3.12) becomes

$$\mu_m(k) \geq p(k)v_m. \tag{3.17}$$

In other words, when the expected marginal social cost at bus m associated with operation under contingency k ($\mu_m(k)$) is greater than or equal to the expected marginal social cost obtained through the use of load shedding ($p(k)v_m$), then load shedding is applied at that bus in response to that contingency.

2. On the other hand, if there is no load shed at bus m following contingency k, that is $l_m(k) = 0$, we have

$$\sigma_m^{lb}(k) \geq 0, \tag{3.18}$$

$$\sigma_m^{ub}(k) = 0. \tag{3.19}$$

Then, (3.12) becomes

$$\mu_m(k) \leq p(k)v_m. \tag{3.20}$$

Hence, if the expected marginal social cost at bus m associated with operation under contingency k ($\mu_m(k)$) is less than or equal to the expected marginal social cost obtained if load shedding were applied ($p(k)v_m$), then this contingency has to be fully covered by scheduling and deploying the appropriate reserve services.

□

The above result is fundamental in the sense that it provides a marginal pricing interpretation of why involuntary load shedding decisions are made. In addition, since the Lagrange multipliers $\boldsymbol{\mu}(k)$ are what make up the prices of energy and security (see Appendix G), the quantities $p(k)v_m$ play a significant role in determining how these prices are set. Said simply, load shedding should be applied at a bus for a given contingency if and only if the marginal expected social cost increment associated with applying load shedding is less than or equal to the marginal expected cost gain obtained through the full scheduling and deployment of reserves.

Clearly, if a contingency is very likely, then its impact on the expected social cost may be more important than unlikely ones thus making the likelihood that this contingency is fully covered with appropriate reserves even more plausible. Likewise, if the value of lost load at a particular bus is higher than at others, then the threshold for which the incremental social cost gain obtained from using load shedding has to be correspondingly higher, making load shedding a less likely response to the spectrum of credible contingencies. It is worth noting here that this result is in line with the conclusions of the earlier works of Siddiqi and Baughman on pricing for differentiated reliability purposes [86, 87].

Expected marginal social costs

Proposition 3.2 (Expected marginal social costs under post-contingency operation). *The expected marginal social costs of security and energy corresponding to operation under contingency k are given respectively by*

$$p(k)\frac{\partial W(k)}{\partial \mathbf{S}} = \left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{S}}\right)^T \boldsymbol{\mu}(k), \tag{3.21}$$

and

$$p(k)\frac{\partial W(k)}{\partial \mathbf{E}} = \left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{E}}\right)^T \boldsymbol{\mu}(k). \tag{3.22}$$

Proof. For contingency k, the first-order KKT optimality condition with respect to the variable sub-vector associated with operation under that contingency, that is $\mathbf{x}(k)$, is

$$p(k)\frac{\partial W(k)}{\partial \mathbf{x}(k)} - \left(\frac{\partial \mathbf{H}(k)}{\partial \mathbf{x}(k)}\right)^T \boldsymbol{\mu}(k) - \left(\frac{\partial \mathbf{G}}{\partial \mathbf{x}(k)}\right)^T \boldsymbol{\sigma} = \mathbf{0}. \tag{3.23}$$

Next, following the reasoning of [64], we perturb the right hand side of the power balance relations under contingency k by the infinitesimal parameter vectors $d\mathbf{E} + d\mathbf{S}$. Here the parameter vector $d\mathbf{E}$ corresponds to an incremental perturbation of both the pre- and the post-contingency power balance equalities, while the parameter vector $d\mathbf{S}$ is a supplemental perturbation applied to the post-contingency power balance constraints only.

Such infinitesimal perturbations, under smoothness assumptions of the functions $W(k)$, $\mathbf{H}(k)$ and $\mathbf{G}$, generate infinitesimal perturbations of the continuous optimization variables, $d\mathbf{x}(k)$ and $d\mathbf{l}(k)$. Since under these perturbations the feasibility of the post-contingency power balance relations must be maintained, we therefore find

$$\begin{aligned} d\mathbf{E} + d\mathbf{S} &= \left(\frac{\partial \mathbf{H}(k)}{\partial \mathbf{x}(k)}\right)^T d\mathbf{x}(k) + \left(\frac{\partial \mathbf{H}(k)}{\partial \mathbf{l}(k)}\right)^T d\mathbf{l}(k) \\ &= \left(\frac{\partial \mathbf{H}'(k)}{\partial \mathbf{x}(k)}\right)^T d\mathbf{x}(k) + \mathbf{I} d\mathbf{l}(k) \\ &= d\mathbf{H}'(k) + d\mathbf{l}(k), \end{aligned} \tag{3.24}$$

where we made use of the results of (3.8) and (3.9). With the help of (3.23) and (3.24), the quantity $p(k)dW(k)$ is re-expressed

$$\begin{aligned} p(k)dW(k) &= \boldsymbol{\mu}^T(k)\frac{\partial \mathbf{H}'(k)}{\partial \mathbf{x}(k)}\,d\mathbf{x}(k) + \boldsymbol{\sigma}^T\frac{\partial \mathbf{G}}{\partial \mathbf{x}(k)}\,d\mathbf{x}(k) \\ &= \boldsymbol{\mu}^T(k)d\mathbf{H}'(k) + \boldsymbol{\sigma}^T d\mathbf{G} \\ &= \boldsymbol{\mu}^T(k)(d\mathbf{E} + d\mathbf{S} - d\mathbf{l}(k)), \end{aligned} \tag{3.25}$$

where we made use of the complementary slackness condition to find that $\boldsymbol{\sigma}^T d\mathbf{G} = 0$.

Rearranging (3.25), we obtain

$$p(k)\frac{\partial W(k)}{\partial \mathbf{S}} = \left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{S}}\right)^T \boldsymbol{\mu}(k), \tag{3.26}$$

$$p(k)\frac{\partial W(k)}{\partial \mathbf{E}} = \left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{E}}\right)^T \boldsymbol{\mu}(k). \tag{3.27}$$

□

In (3.26), the elements of the vector $p(k)\partial W(k)/\partial \mathbf{S}$ are interpreted as the sensitivities of the expected social cost of operation under contingency k to perturbations in the post-contingency power balance relations only. The elements of the matrix $\partial \mathbf{l}(k)/\partial \mathbf{S}$ correspond to the sensitivities of the load shedding variables to these perturbations in the post-contingency power balance relations. Likewise, in (3.27), the vector $p(k)\partial W(k)/\partial \mathbf{E}$ represents the sensitivity of the post-contingency social cost following contingency k due to perturbations of the pre- and post-contingency power balance, while $\partial \mathbf{l}(k)/\partial \mathbf{E}$ is the matrix of load shedding sensitivities to simultaneous perturbations in the pre- and post-contingency power balance.

The most important feature of these relationships, however, is the demonstration of the direct dependance of the post-contingency expected marginal social cost on the sensitivities of the load shedding variables. For instance, it is readily seen that the marginal social cost under contingency k at bus m due to a perturbation of the post-contingency power balance at that bus is given by

$$p(k)\frac{\partial W(k)}{\partial S_m} = \mu_m(k) - \sum_{n=1}^{M} \mu_n(k)\frac{\partial l_n(k)}{\partial S_m}. \tag{3.28}$$

Shuffling terms, assuming that $1 - \partial l_m(k)/\partial S_m \neq 0$, we obtain

$$\mu_m(k) = \left(1 - \frac{\partial l_m(k)}{\partial S_m}\right)^{-1} \left(p(k)\frac{\partial W(k)}{\partial S_m} + \sum_{\substack{n=1 \\ n \neq m}}^{M} \mu_n(k)\frac{\partial l_n(k)}{\partial S_m}\right). \tag{3.29}$$

That is, the dual variable associated with the power balance constraint bus at m following contingency k ($\mu_m(k)$) is equal to the local expected social cost sensitivity $p(k)\partial W(k)/\partial S_m$ adjusted by the marginal effects of load shedding happening locally and at remote buses. In the special case when the perturbation dS_m does not change any of the load shedding variables—that is $\partial l_n(k)/\partial S_m = 0$ for all $n = 1, \ldots, M$—, then the marginal expected social cost boils down to its more "traditional" interpretation

$$\mu_m(k) = p(k)\frac{\partial W(k)}{\partial S_m}, \tag{3.30}$$

which is simply the marginal expected social cost at bus m associated with the scheduling and deployment of reserves corresponding to contingency k.

We also note here that the above comments apply equally to the marginal expected social costs derived from perturbations ($d\mathbf{E}$) of both the pre- and the post-contingency power balance relations. In fact, the results in (3.29) and (3.30) could have been derived equally using the expression for the marginal expected social cost expression associated with the perturbations of the both the pre- and post-contingency power balance equalities (3.27).

Next, we derive a result similar to that of Proposition 3.2 where, this time, we determine the expected marginal social cost of operation under the pre-contingency state.

Proposition 3.3 (Expected marginal social cost under pre-contingency operation). *The expected marginal social cost corresponding to operation under the pre-contingency state is given by*

$$\boldsymbol{\mu}(0) = p(0)\frac{\partial W(0)}{\partial \mathbf{E}}. \tag{3.31}$$

Proof. For the pre-contingency state, the first-order KKT optimality condition with respect to the variable sub-vector $\mathbf{x}(0)$ is

$$p(0)\frac{\partial W(0)}{\partial \mathbf{x}(0)} - \left(\frac{\partial \mathbf{H}(0)}{\partial \mathbf{x}(0)}\right)^T \boldsymbol{\mu}(0) - \left(\frac{\partial \mathbf{G}}{\partial \mathbf{x}(0)}\right)^T \boldsymbol{\sigma} = \mathbf{0}. \tag{3.32}$$

We now perturb the right hand side of the power balance relations in the pre-contingency state by the infinitesimal parameter vector $d\mathbf{E}$. Such infinitesimal perturbation, under smoothness assumptions, generate infinitesimal perturbations of the variables $\mathbf{x}(0)$ only, while the feasibility of the power balance relations must be maintained under such perturbation; therefore, we find that

$$\begin{aligned} d\mathbf{E} &= \left(\frac{\partial \mathbf{H}(0)}{\partial \mathbf{x}(0)}\right)^T d\mathbf{x}(0) \\ &= d\mathbf{H}(0). \end{aligned} \tag{3.33}$$

We can therefore rewrite $p(0)dW(0)$ as

$$\begin{aligned} p(0)dW(0) &= \boldsymbol{\mu}^T(0)\frac{\partial \mathbf{H}(0)}{\partial \mathbf{x}(0)}d\mathbf{x}(0) + \boldsymbol{\sigma}^T\frac{\partial \mathbf{G}}{\partial \mathbf{x}(0)}d\mathbf{x}(0) \\ &= \boldsymbol{\mu}^T(0)d\mathbf{H}(0) + \boldsymbol{\sigma}^T d\mathbf{G} \\ &= \boldsymbol{\mu}^T(0)d\mathbf{E}, \end{aligned} \tag{3.34}$$

where we have made use again of the complementarity condition to find that $\boldsymbol{\sigma}^T d\mathbf{G} = 0$. Rearranging (3.34), we obtain (3.31). □

It is interesting to note the complete decoupling of the pre- and post-contingency states in the computation of the incremental expected social costs under either the pre-contingency or the post-contingency states.

Combining the results of Propositions 3.2 and 3.3 and the pricing principles enounced in Appendix G, we state the following conclusion as a corollary.

Corollary 3.1 (Prices of energy and security). *Under marginal pricing of electricity [3, 125], and following the principles of [64] found in Appendix G, the vectors of prices of energy, $\boldsymbol{\lambda}^E$, and security, $\boldsymbol{\lambda}^S$, are respectively given by*

$$\boldsymbol{\lambda}^E = \frac{\partial W}{\partial \mathbf{E}} = p(0)\frac{\partial W(0)}{\partial \mathbf{E}} + \sum_{k=1}^{K} p(k)\left[\left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{E}}\right)^T\right]^{-1} \frac{\partial W(k)}{\partial \mathbf{E}}, \tag{3.35}$$

and

$$\boldsymbol{\lambda}^S = \frac{\partial W}{\partial \mathbf{S}} = \sum_{k=1}^{K} p(k)\left[\left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{S}}\right)^T\right]^{-1} \frac{\partial W(k)}{\partial \mathbf{S}}, \tag{3.36}$$

if the matrix inverses of $\left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{E}}\right)^T$ *and* $\left(\mathbf{I} - \frac{\partial \mathbf{l}(k)}{\partial \mathbf{S}}\right)^T$ *exist for all* $k = 1, \ldots, K$.

The main feature of Corollary 3.1 is its explicit depiction of the dependance of both of the prices on the sensitivities of the load shedding variables. It is readily seen that if the load shedding sensitivity matrices are zero matrices (meaning that no more load shedding would be applied for the given schedule), then both price vectors correspond to their traditional interpretation of incremental expected social costs. Otherwise, when the load shedding sensitivity matrices are nonzero (meaning that some perturbations of the power balance relation would lead to changes in the optimal load shedding variables), then the incremental expected social cost components, $p(k)\partial W(k)/\partial \mathbf{E}$ and $p(k)\partial W(k)/\partial \mathbf{S}$, are adjusted by the marginal impacts of involuntary load shedding.

Special case: linear formulation

In cases where the formulation of the market-clearing with stochastic security is linear—as in all parts of this dissertation—, the results of Propositions 3.2 and 3.3 as well as those of Corollary 3.1 can be adapted to the specific features of such a formulation. Before deriving these adaptations, we need to consider the following two lemmas. We recall here that discrete variables are assumed fixed at their optimal values as found by the MILP solver. Thus, the following results are derived from the final linear programs solved with the discrete variables fixed at $\mathbf{u}^*$.

Lemma 3.1 (Sensitivity of optimization variables to perturbations in the right-hand side constraint vector of a linear program). *Consider the linear program (LP)*

$$\min_{\mathbf{x} \geq \mathbf{0}} \mathbf{c}^T \mathbf{x}$$

subject to

$$\mathbf{A}\mathbf{x} = \mathbf{b},$$

where any inequalities have been transformed into equalities with the addition of extra slack variables embedded in the variable vector $\mathbf{x}$. *We assume that all redundant constraints have been removed such that the matrix* $\mathbf{A}$ *has full row rank. In addition, let us assume that this problem has a solution* $\mathbf{x}^*$ *whose cost,* $\mathbf{c}^T\mathbf{x}^*$, *is bounded.*

The sensitivity matrix of the optimal solution $\mathbf{x}^*$ *to small enough perturbations* $d\mathbf{b}$ *of* *the right-hand side vector* $\mathbf{b}$ *is given by*

$$\frac{\partial \mathbf{x}^*}{\partial \mathbf{b}} = \begin{bmatrix} \dfrac{\partial \mathbf{x}_B^*}{\partial \mathbf{b}} \\ \dfrac{\partial \mathbf{x}_N^*}{\partial \mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{B}^{-1} \\ \mathbf{0} \end{bmatrix}, \tag{3.37}$$

where $\mathbf{B}$, $\mathbf{x}_B^*$ *and* $\mathbf{x}_N^*$ *are respectively the LP optimal basis matrix and the optimal vectors of basic and non-basic variables.*

Proof. The optimal solution to the LP problem is found from the solution of the linear system of equations $\mathbf{B}\mathbf{x}_B = \mathbf{b}$. Since this linear system has full row and column rank, it is thus invertible such that

$$\mathbf{x}_B^* = \mathbf{B}^{-1}\mathbf{b}, \tag{3.38}$$

and $\mathbf{x}_N^* = \mathbf{0}$.

If a small enough perturbation of the right-hand side vector $d\mathbf{b}$ is applied to this LP—such that the basis matrix and the set of basic variables remain identical—, from (3.38) we see that the basic variables undergo a change $d\mathbf{x}_B = \mathbf{B}^{-1}d\mathbf{b}$ while the non-basic variables stay at their optimal value of zero. Dividing both sides by $d\mathbf{b}$, we obtain (3.37). □

Lemma 3.2 (Post-contingency actions' merit-order under LP market-clearing with stochastic security). *For an LP formulation of the electricity market-clearing with stochastic security, involuntary load shedding at bus m following contingency k is the* last recourse *to be used by the grid operator to restore the power balance.*

Proof. Let us assume that there are available $x_1(k), x_2(k), \ldots, x_N(k)$ bounded reserve resources ($0 \leq x_j(k) \leq \bar{x}_j(k)$ for all $j = 1, \ldots, N$) available to the grid operator to respond to contingency k at bus m; these may include generator- and demand-side resources available locally and/or remotely.[4] These reserve services have marginal social costs which, without loss of generality, satisfy $\beta_1(k) < \beta_2(k) < \cdots < \beta_N(k)$. In addition, the grid operator can use involuntary load shedding (bounded between $0 \leq l_m(k) \leq d_m$) at the rate given by the value of lost load v_m.

Let us assume now the following with respect to the marginal social costs of load shedding and reserves: $\beta_1(k) < \cdots < \beta_J(k) < v_m < \beta_{(J+1)}(k) < \cdots < \beta_N(k)$. We recognize

[4] As seen in Chapter 2 and Appendices B and C, reserve scheduling and deployment actions are bounded by line flow, ramping, capacity and reserve offer limits.

here that for any given bus m the worst case effect of a contingency is the loss of all its pre-contingency power input equal to d_m. Given that the reserve resources are scheduled economically and that the first $j = 1, \ldots, J$ cheaper resources were insufficient to cover the power deficit, that is $\sum_{j \le J} \bar{x}_j(k) < d_m$, involuntary load shedding is the last necessary recourse to close the remaining gap in the power balance since $l_m(k)$ is bounded above by d_m.

Therefore, in this situation we would have $x_j(k) = \bar{x}_j(k)$ for $j = 1, \ldots, J$, $x_j(k) = 0$ for $j = J+1, \ldots, N$ and $l_m(k) = d_m - \sum_{j \le J} \bar{x}_j(k)$. We should note that, as a second consequence in this case, the load shedding variable $l_m(k)$ is a basic variable of the underlying LP.

Otherwise, if $\sum_{j \le J} \bar{x}_j(k) \ge d_m$ then no involuntary load shedding needs to be applied at bus m following contingency k. Lastly, in such cases, the load shedding variable $l_m(k)$ is a non-basic variable of the LP and therefore is equal to zero. $\square$

The above two lemmas lead to the following corollary.

Corollary 3.2 (Complementarity and sensitivity of reserve deployment and load shedding under LP market-clearing with stochastic security). *Assuming that the electricity market-clearing has a solution which entails that the social cost objective is bounded, at bus m under contingency k, we have the following reinterpretation of* (3.24), *where we find that post-contingency reserve deployment and load shedding constitute complementary actions*

$$dE_n + dS_n = \begin{cases} dl_m(k) & \text{if } l_m(k) > 0 \text{ and } n = m, \\ dH_n'(k) & \text{otherwise.} \end{cases} \tag{3.39}$$

As a result, the sensitivities of load shedding at bus m following contingency k with respect to pre- and post-contingency power balance disturbances $d\mathbf{E}$ and $d\mathbf{S}$ are given by

$$\frac{\partial l_m(k)}{\partial E_n} = \frac{\partial l_m(k)}{\partial S_n} = \begin{cases} 1 & \text{if } l_m(k) > 0 \text{ and } n = m, \\ 0 & \text{otherwise.} \end{cases} \tag{3.40}$$

Proof. As a consequence of Lemma 3.2, we have that whenever load shedding is applied at bus m following contingency k, any small enough perturbation of the power balance at that bus will have to be covered by a corresponding change in load shedding there, $dl_m(k)$.

Otherwise, in the case of local perturbations (with $n = m$), other more economical recourses are still available, meaning that the perturbation will have to be covered by corresponding increments in reserve deployment actions, $dH'_m(k)$. In the case when the perturbations are applied at buses $n \neq m$, the result is established by inspection of the second line of (3.24) where it is seen that the local sensitivity of load shedding to remote perturbations is nil (the vector increment of load shedding is multiplied by the identity matrix).

The sensitivity result in (3.40) follows from the above explanation and from Lemma 3.1 as

$$\frac{\partial l_m(k)}{\partial \mathbf{E}} = \frac{\partial l_m(k)}{\partial \mathbf{S}} = \begin{cases} \left(\mathbf{B}^{-1}_{l_m(k)}\right)^T & \text{if } l_m(k) > 0, \\ \mathbf{0} & \text{otherwise,} \end{cases} \tag{3.41}$$

where $\mathbf{B}^{-1}_{l_m(k)}$ is the row of the sensitivity matrix $\mathbf{B}^{-1}$ corresponding to the basic variable $l_m(k)$. From (3.39), it is seen that the elements of the row vector $\mathbf{B}^{-1}_{l_m(k)}$ are all zero except for the m^{th} element whose value has to be equal to 1. □

We now make use of the above results to restate Proposition 3.2 for the LP formulation of the electricity market-clearing problem.

Theorem 3.1 (Marginal expected social cost at bus m under contingency k). *The Lagrange multiplier associated with the post-contingency power balance at bus m, following contingency k, $\mu_m(k)$, assumes the following values depending on the optimal value of the involuntary load shedding variable $l_m(k)$*

$$\mu_m(k) = \begin{cases} p(k)\dfrac{\partial W(k)}{\partial E_m} = p(k)\dfrac{\partial W(k)}{\partial S_m} & \textit{if } l^*_m(k) = 0, \\ p(k)v_m & \textit{if } 0 < l^*_m(k) < d_m, \\ \geq p(k)v_m & \textit{otherwise, when } l^*_m(k) = d_m. \end{cases} \tag{3.42}$$

Proof. Recalling the results of Proposition 3.2, summarized in (3.26) and (3.27), and that of the load shedding sensitivity results of Corollary 3.2, when there is no load shedding applied at bus m under contingency k, we obtain the first case shown in (3.42). Next, the second and third cases are established from Proposition 3.1. □

For most practical purposes, the third case in Theorem 3.1 should never be encountered. This case essentially means that under contingency k, the only recourse used to balance the power would be load shedding. Thus, any extra increment in the power demand at that bus (be it pre- or post-contingency) would have to be covered by more expensive reserve deployment if available (with value $\beta_{(J+1)}(k) > v_m$, violating the assumptions of Lemma 3.2), or would otherwise lead to infeasibility of the LP. Moreover, in this case, the perturbations of the power balance relations would end up modifying the optimal basis of the market-clearing LP because another resource would need to enter the basis so as to balance the load, a situation that was rejected by the assumptions of the above proofs and thus violating again the assumptions of Lemma 3.2. Therefore, it is reasonable to state that

$$\mu_m(k) = \begin{cases} p(k)\dfrac{\partial W(k)}{\partial E_m} = p(k)\dfrac{\partial W(k)}{\partial S_m} & \text{if } l_m^*(k) = 0, \\ p(k)v_m & \text{if } 0 < l_m^*(k) \leq d_m, \end{cases} \tag{3.43}$$

for all contingencies $k = 1, \ldots, K$ and buses $m = 1, \ldots, M$. Finally, we restate Corollary 3.1 as the following theorem.

Theorem 3.2 (Prices of energy and security under LP market-clearing with stochastic security). *Denoting the subset of contingencies for which there is load shedding at bus m as $\mathcal{K}_m$, the marginal prices of energy, λ_m^E, and security, λ_m^S, at that bus are*

$$\lambda_m^E = p(0)\frac{\partial W(0)}{\partial E_m} + \sum_{k \notin \mathcal{K}_m} p(k)\frac{\partial W(k)}{\partial E_m} + \sum_{k \in \mathcal{K}_m} p(k)v_m, \tag{3.44}$$

and

$$\lambda_m^S = \sum_{k \notin \mathcal{K}_m} p(k)\frac{\partial W(k)}{\partial S_m} + \sum_{k \in \mathcal{K}_m} p(k)v_m. \tag{3.45}$$

Proof. The proof is carried out via direct substitution of the results of Theorem 3.1 into those of Corollary 3.1. □

To illustrate some of the abstract results just derived, next we look at a simple numerical example.

3.2.2 Numerical example

Consider the two-bus, two-generator system in Fig. 3.1, whose two buses are linked by a transmission line with a maximal flow capacity of 0.5 per unit. There are inelastic demands of 1 per unit located at both buses. The generators, which are assumed to have infinite capacity and zero lower generation limits, offer to generate power at the rate of a_i dollars per unit and to provide up- and down-spinning reserves at the rates q_i^{up} and q_i^{dn} dollars per unit respectively. These numbers are found in Table 3.1. Moreover, we assume there are no ramping limits affecting the generators, while the reserve offerings are not bounded from above.

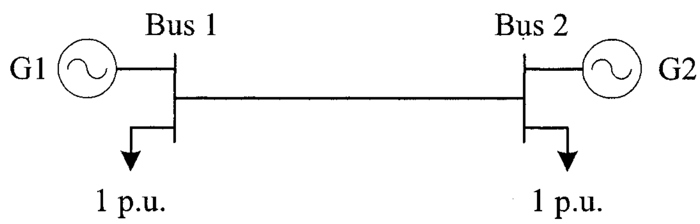

Fig. 3.1 Two-bus, two-generator system

Table 3.1 Generation and reserve offers

		Generator i	
		1	2
a_i	(\$ p.u.)	10	100
q_i^{up}	(\$ p.u.)	20	15
q_i^{dn}	(\$ p.u.)	20	15

Here, we assume that the only credible contingency is the loss of the transmission line ($k = 1$) which has a probability $p(1) = 0.1$. As a result, under post-contingency conditions, the two buses become separate islands.

Next, we examine how the schedules and Lagrange multipliers are affected by the value of lost load (VOLL) at bus 2. For two different VOLL applied at bus 2 (\$200 and \$500 per unit), we resolve the resulting linear market-clearing problem minimizing the expected social costs under the assumption that the VOLL at bus 1 is \$200 per unit. Table 3.2 presents a comparison of the schedules, while Table 3.3 compares the resulting Lagrange multipliers associated with the pre- and post-contingency power balance relations.

By inspection of Table 3.2, we distinguish that under a VOLL of \$200 per unit at bus 2, the amount of involuntary load shedding applied is equal to the power that was originally transferred through the transmission line (0.5 per unit). This is unlike the case under which

Table 3.2 Generation, reserve and load shedding under different VOLL at bus 2

		$v_2 = \$200$ p.u.	$v_2 = \$500$ p.u.
		Pre-contingency	
g_1	(p.u.)	1.5	1.5
r_1^{up}	(p.u.)	0.0	0.0
r_1^{dn}	(p.u.)	0.5	0.5
g_2	(p.u.)	0.5	0.5
r_2^{up}	(p.u.)	0.0	0.5
r_2^{dn}	(p.u.)	0.0	0.0
		Post-contingency	
$g_1(1)$	(p.u.)	1.0	1.0
$l_1(1)$	(p.u.)	0.0	0.0
$g_2(1)$	(p.u.)	0.5	1.0
$l_2(1)$	(p.u.)	0.5	0.0

Table 3.3 Lagrange multipliers associated with the pre- and post-contingency power balances under different VOLL at bus 2

		$v_2 = \$200$ p.u.	$v_2 = \$500$ p.u.
		Pre-contingency	
$\mu_1(0)$	(\$ p.u.)	27.0	27.0
$\mu_2(0)$	(\$ p.u.)	80.0	76.5
		Post-contingency	
$\mu_1(1)$	(\$ p.u.)	-17.0	-17.0
$\mu_2(1)$	(\$ p.u.)	20.0	23.5

the VOLL at bus 2 equals \$500 per unit. In this case, all the power that came from bus 1 through the line is covered by 0.5 per unit of up-spinning reserve provided by generator 2.

The contents of Table 3.3 agree with the theoretical results derived in the previous section. For instance, it is readily seen under the case when $v_2 = \$200$ per unit that $\mu_2(1) = p(1)v_2 = \$20$ per unit, that is the marginal expected cost of load shedding at bus 2. On the other hand, in the case when $v_2 = \$500$ per unit, we have $\mu_2(1) = \$23.5$ per unit, a value less than the marginal expected cost of load shedding $p(1)v_2 = \$50$ per unit. Here the value of the Lagrange multiplier $\mu_2(1)$ is found to be equal to $(1-p(1))q_2^{up}+p(1)a_2$, representing respectively the marginal expected cost of scheduling up-spinning reserve from generator 2 summed with the marginal expected cost of deploying that reserve if the contingency were to happen. Moreover, it is obvious that the sensitivity of the load shedding variable at bus 2 is equal to 1 per unit when $v_2 = \$200$ per unit and that it is nil when $v_2 = \$500$ per unit; that of bus 1 is zero in both VOLL cases.

Finally, we see that under both values of VOLL at bus 2 the prices of energy at buses 1 and 2 are respectively \$10 and \$100 per unit (equal to the respective marginal costs of generation at each bus). The price of security at bus 1 does not differ between the two cases at $-\$17$ per unit.[5] At bus 2, however, the price of security varies under the two distinct VOLL regimes. In the low VOLL case, the consumers are rewarded with a lower expected price of security (\$20.0 versus \$23.5 per unit in the high VOLL case), but run the risk of being cut if the line fails. In the high VOLL case, the consumers at bus 2 are benefiting from a price of security lower than that of the expected social cost of load shedding (at the expected rate of \$50.0 per unit) since it is cheaper to schedule and deploy reserves (at the expected rate of \$23.5 per unit).

3.3 Settlement Issues

In close relation with pricing mechanisms, settlement issues are the object of continuing research and development efforts. In short, settlement schemes are the sets of rules defining how generators are remunerated for the energy they produce and the reserve services they provide whereas, for demands, these rules define how consumers pay for the energy

[5]The negative sign indicates that by raising the demand at bus 1, expected social costs can be reduced because an increase in consumption decreases the need for scheduling down-spinning reserve to counter the line outage.

they consume and how they are remunerated for the reserves they provide. Furthermore, settlement rules are needed to determine who pays (generators only, demands only or both) and how one pays for the necessary reserve services.

Settlement rules for energy are generally straightforward: generators get paid the price of energy times the energy produced, and demands pay the price of energy times the energy consumed [3, 9, 10, 64, 135]. In the case of reserve services—or any ancillary service—, however, it is agreed in the electricity economics community that appropriate and fair settlement rules for reserves are lacking.

So far, the industry has used simple means to settle reserve services through uplift charges levied from the consumers *pro rata* of their energy consumption. The generators and demands that provide reserve services are then remunerated either on a pay-as-bid or on a marginal pricing basis [58–60]. Such schemes are simple and clear; nevertheless, they oversimplify the technical and economic intricacies of reserve and ancillary service provision.

Reserves (and the other ancillary services) possess the characteristics found in so-called *public goods* [125] since, by providing reserves, *all* entities—not just consumers—connected to the grid benefit from them in the form of "security" to sustain the trade of electricity. It was shown in this chapter that security can be priced. However, one question remains: Who should pay for security? The natural and most simplistic answer is to have the consumers pay the entire bill.

Generally, however, there are entities (generators, demands and lines) that impose greater reserve burdens on the system as a whole because of a combination of (i) their high probability of failure; and, (ii) their lumpiness. For example, it is clear that, from a fairness point of view, the U400 nuclear generators of the IEEE Reliability Test System [67], with their large capacity and high outage probabilities, should have to bear a higher responsibility for the costs of security (by having to pay more for reserves) than the small and reliable U50 hydro units in the same system. Several authors have come up with propositions along this line [61, 62, 137]. Further treatment of this issue, however, lies outside the scope of this dissertation.

3.4 Summary

In this chapter, we first summarized the state of affairs in the area of pricing in electricity markets. This review did shed light on the fact that current pricing schemes are not capable of correctly pricing power system security. Yet, we saw how the recent work in [64] provides the necessary framework to achieve this goal.

Taking the Arroyo-Galiana pricing proposition as our starting point, next we specialized it to the problem of electricity market-clearing with stochastic security. We demonstrated a number of theoretical results on the Lagrange multipliers associated with pre- and post-contingency power balance relations, which end up becoming the constituents of the prices of energy and security. Most notably, we showed the economic couplings existing between the probabilities of contingencies, the value of lost load and the marginal social costs of reserve scheduling and deployment. The results were first derived for a general nonlinear market-clearing model and then for linear programming formulations like those found elsewhere in this dissertation. For linear programming formulations, we demonstrated that, for a given contingency, load shedding is used whenever its expected marginal social cost is less than the marginal social cost of scheduling and deploying reserves. We illustrated this in a small numerical example.

Chapter 4

Market-Clearing Under Demand and Wind Generation Uncertainty

The wind flapp'd loose, the wind was still,
Shaken out dead from tree and hill:
I had walk'd on at the wind's will,—
I sat now, for the wind was still.

The Woodspurge
Dante Gabriel Rossetti, 1828–1882

4.1 Introduction

In the last ten years, public environmental awareness to issues of air pollution and climate change has put pressure on governments all over the industrialized world to curb emissions coming especially from the energy and transportation sectors. Following the 1992 Earth Summit in Rio, rounds of international negotiations led to the drafting of the Kyoto protocol which specifies greenhouse gas reduction targets of its signing parties. The nature of the economy and geography of Canada exacerbates the scale of the actions needed to reach the national emission reduction targets set by the Kyoto Protocol [138]. To illustrate the scale of the challenge that Canadians face, it is noteworthy to underline that in 2001 Canada had *per capita* carbon emissions of the order of 19 tons of CO_2 equivalent, coming second among the G8 countries slightly below the United States at 20 tons of CO_2 equivalent [139].

In addition, in 2001 Canada had the largest *per capita* electricity consumption (at 16 000 kilowatt-hours) among the G8 countries, which averaged a *per capita* consumption of 9 500 kilowatt-hours [139].

From these statistics, it is not surprising that the greenhouse gas emissions coming from the electricity sector in Canada are significant. In 2003, the Canadian electricity generation sector accounted for 134 000 kilotons of CO_2 equivalent (18.1% of the country's total emissions) just behind the road transportation sector with 140 000 kilotons of CO_2 equivalent (18.9% of the country's total emissions) [140]. Because of the prominence of the share of the global emissions from the electricity generation sector, there is sustained public and political pressure for increasing the penetration of low carbon electricity generation technologies in Canada [138], technologies which comprise most notably generation installations based on renewable energy sources (wind, solar, biomass, etc.) [65].

In general, these electricity generation resources cannot be scheduled and dispatched in the classical sense because of their intrinsic dependence on constantly-varying weather conditions. We refer the interested reader to [65, 66, 141, 142] for extensive bibliographical reviews and studies about the intermittence of wind power (WP) and to [65] for the case of solar power. Without loss of generality, here we concentrate on WP as it is now the most prominent renewable resource being integrated into existing grids, and since it is probably the most mature renewable generation technology readily available on the market [65, 66, 138]. Other types of non-dispatchable resources may have different modeling particularities; however, the basic principles described below should remain essentially identical.

Large, megawatt-range modern wind turbines generally have mechanisms that attempt to regulate their output as the wind speed varies [65, 66]; however, these local control schemes are designed to extract the maximum power from the wind rather than to respond to a grid operator's centralized dispatch instructions or to the system frequency excursions. As a result, WP generation needs to be backed up by classic hydrothermal generating units to perform the primary, secondary and tertiary regulation actions necessary to maintain reliable grid operation [32–34]. Obviously, in systems where the WP represents a significant proportion of the installed classic hydrothermal generation capacity, like in Denmark and Germany, the regulation needs imposed by the wind power generators may be important and can have significant costs [66, 143–147]. These regulation costs are either socialized among the consumers or they are assumed by the wind producers themselves thus reducing the competitiveness of this energy source.[1] In fact, it is well recognized within the power systems operation community that increasing the level of WP penetration requires a full reassessment of operating methodologies and standards, especially in setting operating reserve requirements governing the primary, secondary and tertiary regulation tasks [143–158].

In addition to the reliability issues, the integration of WP in existing grids has to be made in accordance with the current electricity market structures [143, 145, 147, 159, 160]. Nowadays, WP may be sold in hour-ahead or in real-time electricity markets [143]. Nonetheless, there is no clear agreement on how WP generators should be offering energy in these markets. It is evident, however, that the quality of WP offerings is coupled to the dependability of short-term (24 to 1 hour ahead) wind forecasting techniques [158, 161]. Good and dependable WP prediction techniques are also crucial to grid operators as the WP predictions represent important guides towards scheduling appropriate levels and types of operating reserves needed to perform the different regulation tasks.

In this chapter, we outline how the electricity market-clearing with stochastic security framework developed in the previous chapters can be specialized to perform market-based scheduling of generation, load and tertiary reserve for power systems with uncertain WP generation forecasts. The proposed market-clearing formulation is not restricted to the treatment of the uncertainty in WP generation predictions, but it may also account for load prediction errors. In addition, the formulation includes an operational model for large-scale energy storage systems connected to the grid. These energy storage capabilities, which include mainly pumped-storage hydro, large battery-based and flywheel-based installations, offer interesting hour-to-hour economic arbitrage opportunities and can also facilitate the regulation tasks of the grid operator by buffering short-duration load and generation variations [155, 162–165]. Moreover, the market-clearing model we develop goes beyond similar research and development efforts that deal only with the scheduling of small-scale isolated power systems containing wind and other non-dispatchable resources; see, for example, the work of Handschin *et al.* [166].

We point out that this proposal diverges from previous works on scheduling of power systems with grid-integrated intermittent generation resources and the issues associated

[1]This economic argument is often used by opponents of wind power. Moreover, the opponents claim that the CO_2 abatements brought about by wind get watered down by the need to have hydrothermal backup capacity online operating at lower thermal or hydraulic efficiencies.

with setting reserve needs. In a quest for computational tractability in addressing these scheduling problems, some authors make use of *offline* Monte Carlo simulations [153, 155] while others use analytical work based on empirical data [143, 145, 148–152, 156–158]. The unifying characteristic of the works of these authors is that they all attempt to establish *a priori* system-wide levels of reserves to be provided by the hydrothermal generators to guarantee some desired level of reliability.

Here, however, reserve levels required to accommodate the variations in the WP generation are rather computed *online* based on the principles of electricity market-clearing with stochastic security. The market operator schedules the hydrothermal generators minimizing the expected social cost when the power system operates under (i) a *most likely*, error-free wind power generation/load scenario (alike the pre-contingency state defined in Chapter 2); and, (ii) a spectrum of wind and load scenarios representing the potential errors made in their forecasts (alike contingency scenarios). Also, the proposed scheduling mechanism considers the economic costs of any involuntary load shedding that may be required to balance power under the probable forecasting error scenarios. Lastly, an important assumption here is that in order for the proposed approach to be successful, we presuppose that the primary and secondary regulation tasks [32–34] are sufficient to keep the power system from drifting into instability in the aftermath of disturbances caused by variations in the wind and in the demand.

Case studies illustrating the functioning and the effects of key parameters of the market-clearing formulation then complement the chapter. Among parameters whose effects are of interest are the WP generation penetration level and the size of the energy storage installations.

4.2 Market-Clearing Model Formulation

4.2.1 Demand prediction and uncertainty

Short-term electricity demand prediction tools are numerous and have been the subject of extensive research and development [30, 167]. Here, however, we do not assume the use of any particular technique. Rather, we assume that a prediction technique provides an hour-by-hour sequence of load forecasts (or any time step, as required by the scheduling horizon), $\hat{d}_{mt}$ megawatts for $t = 1, \dots, T$ and demand entities $m = 1, \dots, M$. We note that, for simplicity of exposition here, we ignore transmission network effects.

Since the demand forecasts are generally inaccurate, we model the forecast errors as zero-mean normally-distributed random variables (RV) θ_{mt} with (predicted) standard deviation σ_{mt} megawatts for $t = 1, \ldots, T$ and $m = 1, \ldots, M$. The normality assumption of the demand forecast error is standard in the literature [46, 152, 167]. It is justified through the wide diversity of the electricity demand across geographical areas and consumer classes combined with an invocation of the central limit theorem [168]. In an electricity market setting, a number of caveats about demand forecasts and forecast error must be addressed. First, demand forecasts are performed by the load-serving entities $m = 1, \ldots, M$, which then bid accordingly in the market on behalf of their consumers. As a result, the system-wide demand forecast is obtained as a by-product of demand-side bidding in the forward (*e.g.* day-ahead) electricity market.[2] That is, the forecast sequences $\hat{d}_{mt}$ for $t = 1, \ldots, T$ and $m = 1, \ldots, M$ are determined through the benefit functions of demands, which reflect consumers' price elasticities. Yet, we assume here that the error in the demand forecast is independent of the consumer benefit functions. This assumption is justified because forecast errors are caused generally by uncontrollable factors exogenous to the demand agents, weather being one of the prime examples [167].

4.2.2 Wind power prediction and uncertainty

With the increasing interest for integrating wind into existing grids, WP generation prediction is currently a subject of extensive ongoing research and development [66,147,158,161].

Nevertheless, here we assume—like in the case of the demand—that we already have a prediction tool that provides an hour-by-hour (or any time step, as required by the scheduling horizon) system aggregate WP generation forecast sequence, $\hat{w}_t$ megawatts for $t = 1, \ldots, T$. We likewise model the forecast errors as zero-mean normally-distributed RVs θ_{wt} with (predicted) standard deviation σ_{wt} megawatts for $t = 1, \ldots, T$.

Statistical models for wind speeds at specific locations do not fit normal distributions but rather Rayleigh distributions [65,66]. In addition, this reality combined with the wind turbines' nonlinear wind speed-to-power output relationships result in that the probability distributions of the power output of *individual* wind generators are not normal. However, like in the case of the demand, the large number and the geographical dispersion of the

[2]Note, however, that generally grid operators also perform demand forecasts for reliability reasons.

wind turbines permit the invocation of the central limit theorem [168] to justify the normality assumption of the prediction error. Of course, there may be cases where the poor geographical diversity of the wind-based generation capacity cannot justify making this assumption. The treatment of such complications, however, is beyond the scope of this dissertation.

4.2.3 Demand and wind power: the concept of net load

From the demand and wind power generation forecasts, it is possible to define what is generally termed the *net load* forecast $\hat{n}_t$ and its associated forecast error RV θ_{nt} [65,66,146]. Given that both the discrete time demand and WP generation random processes have similar frequency spectra, during some time period t, we define the net load forecast $\hat{n}_t$ as the difference between the demand and the WP generation forecasts

$$\hat{n}_t = \sum_{m=1}^{M} \hat{d}_{mt} - \hat{w}_t. \tag{4.1}$$

Since it is generally assumed that forecast errors are uncorrelated normal RVs, then the standard deviation of the forecast error associated with the net load σ_{nt} is calculated as [66, 146, 158, 168]

$$\sigma_{nt} = \sqrt{\sum_{m=1}^{M} \sigma_{mt}^2 + \sigma_{wt}^2}, \tag{4.2}$$

for $t = 1, \ldots, T$.

In the remaining parts of this chapter, to simplify the notation we will use the net load concept in formulating the electricity market-clearing problem with WP generation and demand uncertainty. In doing so, the net load forecast $\hat{n}_t$ will be used with its zero-mean normally-distributed error RV θ_{nt}, which has a standard deviation σ_{nt}.

4.2.4 General formulation

Continuous forecast uncertainty model

Under the continuous normally-distributed net load forecast model, the scheduling problem at hand is a stochastic optimal control problem [169, 170] minimizing a function measuring the expected social cost of operating under the possible range of realizations of the net load error RV $\theta_{nt} \in \mathbb{R}$, over the discrete time intervals $t = 1, \ldots, T$. Mathematically, this translates to

$$\min_{\mathbf{u},\mathbf{x}} \sum_{t=1}^{T} \int_{-\infty}^{\infty} W(\mathbf{u}_t(\theta_{nt}), \mathbf{x}_t(\theta_{nt}))\, dF(\theta_{nt}) \tag{4.3}$$

subject to

$$\mathbf{H}(\mathbf{u}_t(\theta_{nt}), \mathbf{x}_t(\theta_{nt}), \theta_{nt}) = \mathbf{0}; \qquad t = 1, \ldots, T, \tag{4.4}$$

$$\mathbf{G}(\mathbf{u}_t(\theta_{nt}), \mathbf{x}_t(\theta_{nt}), \theta_{nt}) \geq \mathbf{0}; \qquad t = 1, \ldots, T, \tag{4.5}$$

where $\mathbf{u}_t(\cdot)$ and $\mathbf{x}_t(\cdot)$ are control decision vectors defined respectively in the space of functions $\mathbf{u}_t : \mathbb{R} \rightarrow \mathbb{B}^P$ and $\mathbf{x}_t : \mathbb{R} \rightarrow \mathbb{R}^Q$ for $t = 1, \ldots, T$. Also, the scalar function $W : \mathbb{B}^P \times \mathbb{R}^Q \rightarrow \mathbb{R}$ measures the social cost—that includes hydrothermal generation operation and reserve costs less the demand-side benefits—, and $dF(\theta_{nt})$ denotes the differential of the cumulative probability distribution function of the net load error RV during period t. We also note that as the time horizon of the electricity market-clearing problem is short here (*i.e.* up to 24 hours), the objective function does not include a discounting term. The feasible set of the scheduling problem is described abstractly by the vector equality (4.4) and the vector inequality (4.5) constraints, both of which are parameterized by the realizations of the net load error RV θ_{nt} for $t = 1, \ldots, T$. An optimal solution[3] of (4.3)–(4.5), $(\mathbf{u}^*(\boldsymbol{\theta}_n), \mathbf{x}^*(\boldsymbol{\theta}_n))$, is a functional map of the net load that minimizes the expected social cost and that respects all equality and inequality constraints of the scheduling problem.

Formulating and solving the discrete-time stochastic optimal control problem (4.3)–(4.5) are very difficult tasks given the computing power available today and probably in the future [170]. Therefore, it is more reasonable to consider an approximation of (4.3)–(4.5) whereby the continuous probability distribution of the net load error RV is discretized in a number of representative "slices" [85, 170].

Discrete forecast uncertainty model

Fig. 4.1 shows an example of such a discretization of the continuous probability distribution function of the net load forecast error where there are seven intervals centered on the mean and where each of the intervals are one net load forecast error standard deviation (σ_{nt})-wide.

[3]This assumes that at least one solution exists.

Obviously, other slicing designs can be adopted with more intervals to improve the quality of the approximation at the expense of a larger problem size. Likewise, uneven slicing patterns can be used whereby more intervals are clustered closer to the mean and fewer are used to model the tails of the distribution. Advanced techniques for approximating the distributions of continuous RVs with discrete ones have been developed and used especially in mathematical finance; see [171–174] for example.

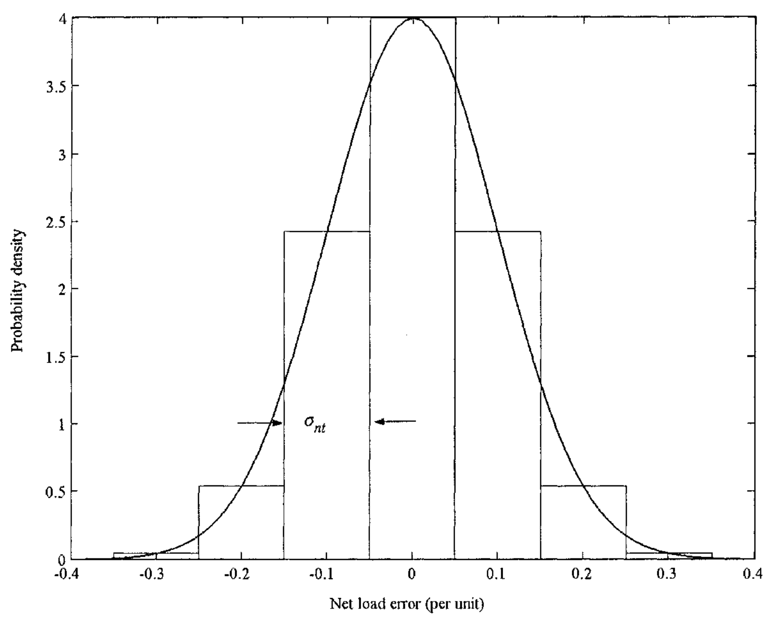

Fig. 4.1 Typical probability distribution of the net load forecast error

Specifically, the slicing process yields a sequence of pairs $(\theta_{nt}(j), \nu_t(j))$ for $j = 1, \ldots, J$ and $t = 1, \ldots, T$. For each j, $\theta_{nt}(j)$ is defined at the center of its respective interval while its corresponding probability is evaluated using standard techniques. For example, in Fig. 4.1 the discrete realizations of the net load error RV are $\theta_{nt}(1) = -0.3$ per unit, $\theta_{nt}(2) = -0.2$ per unit and so on. Their corresponding interval probabilities $\nu_t(j)$ are calculated from first principles as shown in Appendix H.1.

Net load scenarios

In the stochastic optimal control problem (4.3)–(4.5), the optimization process is conducted over the infinite number of discrete-time trajectories of the net load error RV. Given now the assumption that the net load forecast error can only adopt a finite number of values during each time period, the above optimization problem will therefore be conducted over a finite number of such trajectories. This observation leads to the following definitions.

Definition 4.1 (Node). A *node* j, where $j \in \{1, \ldots, J\}$, represents one of the possible discrete realizations of the net load error RV.

Definition 4.2 (Net load forecast error scenario). A *net load forecast error scenario* $k \in \{1, \ldots, K\}$, denoted as $\mathcal{S}_k$, is an ordered sequence of nodes $\{j_{k1}, j_{k2}, \ldots, j_{kT}\}$ defining a trajectory of discrete realizations of the net load error RV over the scheduling horizon.

Definition 4.3 (Error-free scenario). The *error-free scenario* $\mathcal{S}_{\hat{k}} = \{j_{\hat{k}1}, j_{\hat{k}2}, \ldots, j_{\hat{k}T}\}$ is the scenario for which the realization of the net load error RV is equal to zero for all $t = 1, \ldots, T$. The concept of the error-free scenario parallels that of the pre-contingency scenario defined in Chapter 2.

Definition 4.4 (Scenario tree). A *scenario tree* $\mathcal{T}$ is a collection of net load forecast error scenarios, that is $\mathcal{T} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_K\}$. A scenario tree must always contain at least the error-free scenario, $\mathcal{S}_{\hat{k}}$.

Associated with each scenario $\mathcal{S}_k$, there is a time-indexed sequence of probabilities $p_t(k)$, which is calculated as shown in Appendix H.2. Moreover, associated with each scenario $\mathcal{S}_k$, there is a pre-calculated sequence of realizations of the net load error RV $\theta_{nt}(k)$.[4] From these sequences, the discrete-time stochastic optimal control problem (4.3)–(4.5) can be reformulated as

$$\min_{\mathbf{u},\mathbf{x}} \sum_{t=1}^{T} \sum_{k=1}^{K} p_t(k)\, W(\mathbf{u}_t(k), \mathbf{x}_t(k)) \tag{4.6}$$

subject to

$$\mathbf{H}(\mathbf{u}_t(k), \mathbf{x}_t(k), \theta_{nt}(k)) = \mathbf{0}; \qquad t = 1, \ldots, T,\ k = 1, \ldots, K, \tag{4.7}$$

$$\mathbf{G}(\mathbf{u}_t(k), \mathbf{x}_t(k), \theta_{nt}(k)) \geq \mathbf{0}; \qquad t = 1, \ldots, T,\ k = 1, \ldots, K, \tag{4.8}$$

where the mappings defined previously retained their respective domain and range spaces. This time, an optimal solution[5] to the problem (4.6)–(4.8), $(\mathbf{u}^*(\mathbf{k}), \mathbf{x}^*(\mathbf{k}))$, is no longer a

[4]To avoid overloading the notation here, we let $\theta_{nt}(k) \equiv \theta_{nt}(j_{kt})$. It is understood that under scenario $\mathcal{S}_k$, the realization j_{kt} of the net load error RV occurs during period t.

[5]This assumes that at least one solution exists.

set of functional maps defined over the real line; it is rather a set of functional maps defined over the finite number of realizations of the net load error random variable.

As mentioned before, this simplification is a necessary condition for this problem to be computationally tractable. Any arbitrary level of accuracy could be attained by shrinking the width of the slices of the net load forecast error probability distribution and letting the number of slices approach infinity. Nevertheless, this has to be done at the expense of a loss in computational tractability.

Statistical studies of inter-hour wind generation variations in Scandinavia [141, 146] point out that, most of the time, these variations remain within ±5–10% of the installed wind capacity. Likewise, inter-hour demand deviations from the load forecast are usually relatively well bounded. As a result, in formulating the scheduling problem (4.6)–(4.8), it may be conceivable—and, in fact, necessary—to optimize over a scenario tree which is made up of those scenarios that do not contain very "unlikely" inter-period transitions. Such simplifications should render the problem more computationally tractable, given that the total number of nodes and the number of scenarios in a scenario tree both grow exponentially with the duration of the scheduling horizon as J^T.[6] In simplifying the scenario tree, some *ad hoc* scenario rejection techniques based on empirical evidence could be used. The other systematic techniques applicable to generic stochastic optimization problems mentioned in Chapter 2 [105, 121–123, 175] should be investigated as well.

Fig. 4.2 shows an example for a case with $J = 3$ defining "Low", "As predicted" and "High" net load forecast error slices evolving over $T = 2$ time periods. In Fig. 4.2, all transitions are allowed, leading to a nine-scenario scenario tree. Moreover, here the error-free scenario is easily identified as the sequence {As predicted, As predicted}.

Now that the scheduling problem may be specified for a scenario tree $\mathcal{T}$ that may not contain all possible scenarios, (4.6)–(4.8) is rewritten using shorthand notation

$$\min_{\mathbf{u},\mathbf{x}} \sum_{(k,t)\in\mathcal{T}} p_t(k)\, W(\mathbf{u}_t(k), \mathbf{x}_t(k)) \tag{4.9}$$

[6]The scenario tree corresponding to the seven-interval net load forecast error probability distribution shown in Fig. 4.1 that runs over a scheduling horizon of 24 hours contains over 1.9×10^{20} scenarios if all possible inter-hour interval-to-interval transitions are allowed.

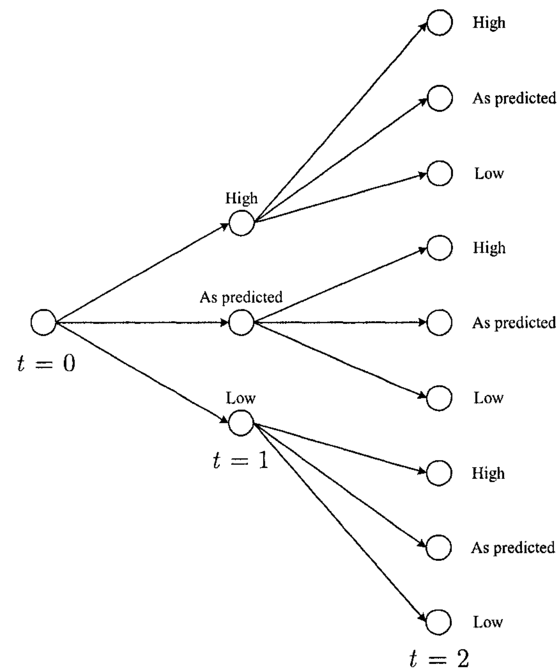

Fig. 4.2 Net load forecast error scenario tree example

subject to

$$\mathbf{H}(\mathbf{u}_t(k), \mathbf{x}_t(k), \theta_{nt}(k)) = \mathbf{0}; \qquad \forall\, (k,t) \in \mathcal{T}, \tag{4.10}$$

$$\mathbf{G}(\mathbf{u}_t(k), \mathbf{x}_t(k), \theta_{nt}(k)) \geq \mathbf{0}; \qquad \forall\, (k,t) \in \mathcal{T}. \tag{4.11}$$

In the following subsections, details specific to the above stochastic optimization problem are addressed. Namely, the characterization of its social cost objective, found above in (4.9), as well as that of its feasible set, described abstractly in (4.10) and (4.11). Among specific aspects, we examine those related to the inclusion of energy storage, wind energy spillage, involuntary load shedding and the particularities of reserve determination.

4.2.5 Meeting the power balance: energy storage, wind power spillage and involuntary load shedding

It was mentioned in the Introduction of this chapter that the use of energy storage technologies is a promising means to help balancing power by smoothing out WP generation and load variations as well as by providing inter-period economic arbitrage capabilities. Here the storage installations considered are of the centralized type, connected at the transmission level of the grid. These are "macro" energy storage equipment capable of storing energy in the range of a few megawatt-hours. These types of installations contrast with the "micro" energy storage systems directly coupled to individual WP generators or wind farms [162]. The micro energy storage solutions are attractive as they can improve the minute-by-minute regulation of the output of any single or collection of wind turbines. They are also advantageous in terms of reliability because the loss of one or several small storage systems of a few kilowatt-hours each may not be as critical as the outage of a single large storage facility. Yet massive, centralized energy storage installations (*e.g.* large pumped-storage hydro stations) have the advantage, from the point of view of the grid operator, of being fully dispatchable like any hydrothermal unit and, most importantly, they can provide the hour-to-hour energy and economic arbitrage opportunities unlike the small local storage systems only capable of buffering variations with short time constants.

In this section, we formulate two (one lossless, one lossy) generic centralized energy storage models. By generic here, it is meant that there are no *a priori* assumptions made about the storage technology being modeled. The parameters characterizing the generic storage technology are, for both lossless and lossy models, [155]:

- Maximum storage capacity, $e^{\max}$ megawatt-hours;
- Maximum charging/discharging rate, $z^{\max}$ megawatts;
- Maximum storage ramping rate, $\bar{z}^{\max}$ megawatts per hour.

For the lossy model, two more parameters need to be considered:

- Charging efficiency, $\eta^c \in (0, 1)$ per unit;
- Discharging efficiency, $\eta^d \in (0, 1)$ per unit.

In cases when realizations of the net load error RV assume extreme values—whether high or low, although with low probabilities—, the deployment of the available energy storage capabilities, that of the hydrothermal generation reserves and of the voluntary demand-side adjustments may not be sufficient or economically justified to meet the power balance. As a result, for some net load scenarios it may be justified to either (i) "spill" off wind power generation at no cost when the net load is low; or otherwise, (ii) perform involuntary load shedding at a cost, given by the value of lost load, when the net load is high. Note that, for the sake of completeness, it should be mentioned that in cases where there are no energy storage installations connected to the grid, the power balance models defined below do not include any of the variables associated with energy storage.

Market-clearing model with lossless energy storage

Power balance

For all $(k, t) \in \mathcal{T}$, the power balance is given by

$$\sum_{i=1}^{I} g_{it}(k) + \sum_{m=1}^{M} l_{mt}(k) + z_t(k) - s_t(k) = \sum_{m=1}^{M} d_{mt}(k) - \hat{w}_t + \theta_{nt}(k). \tag{4.12}$$

Here we recall that the variables $g_{it}(k)$ represent the power output of generator i during period t under scenario $\mathcal{S}_k$ of the net load error RV.

Voluntary and involuntary load shedding

In (4.12), the variables $l_{mt}(k)$ correspond to involuntary load shedding that could be applied, where of necessity we impose

$$0 \leq l_{mt}(k) \leq \sum_{m=1}^{M} [d_{mt}(k) + \theta_{mt}(k)]; \quad k \neq \hat{k}, \tag{4.13}$$

where $\theta_{mt}(k)$ is the forecast error associated with demand m during period t under scenario $\mathcal{S}_k$. Again, (4.13) is defined for all time intervals and realizations of the net load RV forming the scenario tree $\mathcal{T}$, except for the error-free scenario $\mathcal{S}_{\hat{k}}$ for which we require that $l_{mt}(\hat{k}) = 0$. This is just like in Chapter 2, where we imposed that the involuntary load shedding should be nil under the pre-contingency scenario.

In (4.12) and (4.13), the demand-side variables d_{mt} are augmented with the scenario index k to model the demand-side adjustments that may be commanded by the grid operator as part of their voluntary response to errors in the combined wind power and demand forecasts. We point out that in the case of the error-free scenario, the demand variables are given by $d_{mt}(\hat{k}) = \hat{d}_{mt}$, where $\hat{d}_{mt}$ is the forecasted demand. Here the forecasted wind power generation $\hat{w}_t$ is not indexed by k since, unlike the demand, it may not be controlled directly through voluntary adjustments.

Rate of charge/discharge

Next in (4.12), the variable $z_t(k)$ represents the rate of energy discharge coming from the aggregated energy storage installations over the power system, during interval t and under scenario $\mathcal{S}_k$. This variable is a continuous variable satisfying, for all $(k,t) \in \mathcal{T}$

$$-z^{\max} \leq z_t(k) \leq z^{\max}, \tag{4.14}$$

where $z^{\max} > 0$.[7] Thus, when $z_t(k) > 0$ the energy storage installations are collectively releasing energy into grid; otherwise, when $z_t(k) < 0$ energy is taken out of the grid and being stored. Obviously, when $z_t(k) = 0$ no energy is being stored into nor released from the energy storage installations.

Some ramping limitations may be imposed on the charging/discharging rate so as to model possible time-based restrictions in changing the operating state of the storage installations. Thus, for all $(k,t) \in \mathcal{T}$

$$z_t(k) \leq z_{(t-1)}(k) + \bar{z}^{\max}, \tag{4.15}$$

$$z_t(k) \geq z_{(t-1)}(k) - \bar{z}^{\max}, \tag{4.16}$$

where $\bar{z}^{\max} > 0$. These restrictions apply especially to pumped-hydro storage facilities that cannot go through very fast operating mode changes—that is, from pumping to generating and vice versa.

In addition, we point out here that there is no need to explicitly distinguish between the charging and discharging modes of operation of the energy storage installations with an extra state variable. We will see in a following subsection that under a lossy storage model, it is necessary to distinguish between the modes of operation of the storage installations.[8] This saving clearly represents a fair computational advantage of the lossless storage model over the lossy one, as will be seen later.

[7] We note that the upper and lower limits on the rate of charging/discharging of the storage installations need not be identical.

[8] Even though their storage model is lossless, the authors of [176] do distinguish between the modes of operation of a storage system. This distinction is rendered necessary by the specific storage technology, namely battery storage, and the level of modeling detail used by the authors.

Spillage of wind power generation

Further in (4.12), the variable $s_t(k)$ models the rate at which wind power generation is curtailed or "spilled" during period t under scenario $\mathcal{S}_k$. Much like the involuntary load shedding term, the WP generation spillage is bounded from below by zero and from above by the actual WP generation

$$0 \leq s_t(k) \leq \hat{w}_t + \theta_{wt}(k), \tag{4.17}$$

for all $(k, t) \in \mathcal{T}$. Here, we recall that $\theta_{wt}(k)$ is the forecast error associated with the WP generation during period t under scenario $\mathcal{S}_k$. We also note that unlike the demand, we allow for WP to be curtailed under the error-free scenario.

The use of spillage may seem counterintuitive since the WP power input into the grid is costless. However, under extreme—and for the most part very improbable—situations under which the WP generation is very high and the demand is very low, the expected social cost may be lower if the wind energy is simply spilled. In such cases, the expected costs of the required down-spinning reserve services coming from hydrothermal units and demands can easily outweigh the benefits of the free, but improbable, high wind. We note that the wind energy spillage is generally obtained through active and passive mechanical controls of the wind turbines' blade pitch angle and nacelle yaw angle [65].

Energy storage balance and capacity limits

Under scenario $\mathcal{S}_k$, the energy stored at the end of time period t $e_t(k)$ equals the initial energy stored at the beginning of the period $e_{(t-1)}(k)$ less any energy that is returned back to the grid, $z_t(k)\,\Delta$. Mathematically, this translates into

$$e_t(k) = e_{(t-1)}(k) - z_t(k)\,\Delta, \tag{4.18}$$

where we recall that the quantity Δ represents the length of the time interval between t and $t+1$. Of course, this requirement applies to all pairs (k, t) forming the scenario tree $\mathcal{T}$.

In addition to the energy balance requirement, the energy storage can be charged only within its capacity limits

$$0 \leq e_t(k) \leq e^{\max}, \tag{4.19}$$

where, without loss of generality, we assume that the lower limit on the energy stored is nil and that $e^{\max} > 0$ for all $(k,t) \in \mathcal{T}$.

Energy storage endpoint constraints

At the beginning of the scheduling horizon, $t = 0$, the energy storage installations are assumed to have some initial energy stored $e_0 \geq 0$

$$e_t(k) = e_0, \tag{4.20}$$

for all scenarios $\mathcal{S}_k$.

The initial energy stored e_0 is an exogenous parameter which has to be estimated by the grid operator. One possible strategy here is to estimate e_0 from the expected value of the energy stored in the last period ($t = T$) of the previous scheduling horizon; that is

$$e_0 = \hat{e}_T = \sum_{k:(k,T)\in\mathcal{T}} p_T(k)\, e_T(k). \tag{4.21}$$

This leads into addressing whether or not constraints on the energy stored at the end of the scheduling horizon should be imposed. Given that the market-clearing formulation minimizes the social cost over a finite-duration scheduling horizon, there is no *explicit* incentive to keep any energy stored at the end of a specific horizon. This is because the operating cost during the last few periods of the scheduling horizon can be reduced by discharging the "free" available energy stored. This energy has the potential to back off some of the more expensive thermal generators, which can therefore reduce operational costs.

This result reflects the myopic nature of finite-horizon optimal control problems. Knowing that the market-clearing problem is to be solved again for the following scheduling horizon, the initial conditions used to compute the next schedule are based on the end-state of the current schedule. Thus, one possibility here is to impose a lower bound on the amount of energy stored at the end of each of the scheduling horizons. An obvious way to achieve this goal is to limit from below the expected value of the energy stored at $t = T$, as calculated in (4.21).

An alternative to the imposition of a lower bound on the expected value of the energy stored at $t = T$ is to assign a monetary value to this energy, and to lump it into the social cost objective function. Likewise, it is also possible to combine both approaches using a lower bound and the value of the energy stored.

Market-clearing model with lossy energy storage

Realistic energy storage systems incur losses during their charging and discharging processes [155]. Moreover, the charging and discharging processes may have distinct efficiencies. If storage losses are to be accounted for, these characteristics therefore require a reformulation of the energy storage and power balance models developed above.

It will be shown below that the lossy energy storage model is computationally more demanding compared to its lossless counterpart. In fact, one could argue that including losses associated with the energy storage installations is simply not warranted. First, since the charging/discharging rates of energy storage installations are generally small with respect to the combined installed hydrothermal and wind power capacity, their associated losses should be proportionally small. In addition, since transmission losses—which in reality should be much more important than those associated with the energy storage installations—are ignored in the market-clearing model, it can be argued that storage losses should be ignored as well. Notwithstanding its complexity, but rather for the sake of modeling completeness, the lossy energy storage is described next.

Power balance

For all $(k,t) \in \mathcal{T}$, the power balance under lossy storage is given by

$$\sum_{i=1}^{I} g_{it}(k) + \sum_{m=1}^{M} l_{mt}(k) + \eta^d z_t^d(k) - z_t^c(k) - s_t(k) = \sum_{m=1}^{M} d_{mt}(k) - \hat{w}_t + \theta_{nt}(k), \qquad (4.22)$$

where, specifically here, we distinguish between the discharging, $z_t^d(k)$, and the charging, $z_t^c(k)$, rates of the energy storage installations. We note as well that the energy taken out of the storage installations is converted back into electricity at an efficiency η^d.

Voluntary and involuntary load shedding and spillage

The bounds developed previously in the lossless model for the involuntary load shedding and for the spillage variables, (4.13) and (4.17) respectively, apply equally here.

Rate of charge/discharge

The discharging and charging rates $z_t^d(k)$ and $z_t^c(k)$ are also bounded for all $(k,t) \in \mathcal{T}$

$$0 \leq z_t^d(k) \leq z^{\max}, \tag{4.23}$$

$$0 \leq z_t^c(k) \leq z^{\max}, \tag{4.24}$$

where, for simplicity here, we assume that both upper bounds on the discharging and charging rates are identical and positive. In addition, since the charging and discharging operating modes are mutually exclusive, that is $z_t^d(k)\, z_t^c(k) = 0$ for all $(k,t) \in \mathcal{T}$, it is required to modify (4.23) and (4.24) such that

$$0 \leq z_t^d(k) \leq (1 - y_t(k))\, z^{\max}, \tag{4.25}$$

$$0 \leq z_t^c(k) \leq y_t(k)\, z^{\max}, \tag{4.26}$$

where the variable $y_t(k) \in \mathbb{B}$ and satisfies for all $(k,t) \in \mathcal{T}$

$$y_t(k) = \begin{cases} 1 & \text{if charging,} \\ 0 & \text{otherwise.} \end{cases} \tag{4.27}$$

By requiring an additional binary variable per time period and per scenario, the lossy energy storage model puts a further computational burden on the solution process of the electricity market-clearing problem. Clearly here, the formulation of a succinct scenario tree is paramount to the good performance of the solution process by a mixed-integer linear programming solver.

Lastly, extra constraints, similar to (4.15) and (4.16), are needed to constrain the inter-period changes in both the charging and the discharging rate variables.

Energy storage balance and capacity limits

So far, we have included only the discharging losses in the model via the power balance equation (4.22). The modeling of the charging losses is captured by the energy storage balance. In a way similar to (4.18), the energy stored $e_t(k)$ at the end of time period t

under scenario $\mathcal{S}_k$ is given by

$$e_t(k) = e_{(t-1)}(k) + \eta^c \, z_t^c(k) \, \Delta - z_t^d(k) \, \Delta. \tag{4.28}$$

The power taken away from the grid, $z_t^c(k)$, is stored at an efficiency η^c. The energy stored in the lossy model is also bounded as shown in (4.19). Lastly here, as always these constraints apply for all $(k, t) \in \mathcal{T}$.

Energy storage endpoint constraints

The previous discussions of endpoint effects on the energy stored at times $t = 0$ and $t = T$ for the lossless model apply equally here.

4.2.6 Reserve determination constraints

Like in the case of market-clearing with equipment contingencies described in Chapter 2, the notion of reserve used here is different from what is the current industry definition. In the way exposed in Appendix C, the levels of generation-side reserve (up-spinning and down-spinning)[9] assigned to particular hydrothermal generators satisfy

$$0 \leq r_{it}^{up} \leq r_{it}^{up\ \max} u_{it}; \qquad i = 1, \ldots, I, \; t = 1, \ldots, T, \tag{4.29}$$

$$0 \leq r_{it}^{dn} \leq r_{it}^{dn\ \max} u_{it}; \qquad i = 1, \ldots, I, \; t = 1, \ldots, T, \tag{4.30}$$

and

$$r_{it}^{up} \geq g_{it}(k) - g_{it}(\hat{k}); \qquad i = 1, \ldots, I, \; \forall \, (k, t) \in \mathcal{T}, \tag{4.31}$$

$$r_{it}^{dn} \geq g_{it}(\hat{k}) - g_{it}(k); \qquad i = 1, \ldots, I, \; \forall \, (k, t) \in \mathcal{T}, \tag{4.32}$$

where the variables $g_{it}(\hat{k})$ represent the generation levels under the error-free net load scenario. However, unlike in Appendix C, here the lower bounds (4.31) and (4.32) are not applied for all credible contingency k occurring during period τ, but rather for all realizations of the net load forecast error scenarios $\mathcal{S}_k$ and time periods t forming the

[9]Without loss of generality, we do not consider non-spinning reserve services here. This means that $u_{it}(k) = u_{it}(\hat{k})$ for all $(k, t) \in \mathcal{T}$. Thus, for notational simplicity, the binary unit commitment variables are not overloaded with the extra argument (k).

scenario tree $\mathcal{T}$.

The levels of demand-side reserves assigned to the demand entities are defined along the same lines

$$0 \leq r_{mt}^{up} \leq r_{mt}^{up\,\max}; \qquad m = 1, \ldots, M, \; t = 1, \ldots, T, \tag{4.33}$$
$$0 \leq r_{mt}^{dn} \leq r_{mt}^{dn\,\max}; \qquad m = 1, \ldots, M, \; t = 1, \ldots, T, \tag{4.34}$$

and

$$r_{mt}^{up} \geq d_{mt}(\hat{k}) - d_{mt}(k); \qquad m = 1, \ldots, M, \; \forall\,(k,t) \in \mathcal{T}, \tag{4.35}$$
$$r_{mt}^{dn} \geq d_{mt}(k) - d_{mt}(\hat{k}); \qquad m = 1, \ldots, M, \; \forall\,(k,t) \in \mathcal{T}. \tag{4.36}$$

Like for the generation, the variables $d_{mt}(\hat{k})$ correspond to the demand levels associated with the error-free scenario. Here demands that offer reserves may be called in by the grid operator to modify their consumption so to respond to specific realizations of the net load error random variable.

4.2.7 Hydrothermal generation- and demand-side operational constraints

Appendix B thoroughly details the feasible operational regions applying to both the hydrothermal generators and the demands. In the current context, some modifications of the ramping limitations of the hydrothermal generators must be made to account for the different uncertainty model used here; likewise, some modifications must be made with respect to the demand-side elasticity limits.

Generation-side constraints

Unlike in Chapter 2, where the uncertainty is revealed only once when contingency k happens during period τ, here the uncertainty in the net load is revealed at each time period of the scheduling horizon. As a result, the inter-temporal constraints affecting the hydrothermal generators must be modified accordingly to reflect this different uncertainty structure.

Since we assume that the unit commitment variables u_{it} are set independently of the realizations of the net load error RV, then the minimum up- and down-time constraints applying to the hydrothermal generators are identical to those given in Appendix B. The up- and down-going ramping limitations, however, must account for the fact that the realization of the net load forecast error RV differs from one time period to the next according to the scenario followed.

Hence, for all scenarios $\mathcal{S}_k$, during interval t the upper bounds on $g_{it}(k)$ are given by

$$g_{it}(k) \leq g_{i(t-1)}(k) + R_i^{up} u_{i(t-1)} + R_i^{su}(u_{it} - u_{i(t-1)}) + g_i^{\max}(1 - u_{it}); \quad t = 2, \ldots, T, \quad (4.37)$$

$$g_{it}(k) \leq g_{i0} + R_i^{up} u_{i0} + R_i^{su}(u_{it} - u_{i0}) + g_i^{\max}(1 - u_{it}); \quad t = 1, \quad (4.38)$$

for all hydrothermal generators $i = 1, \ldots, I$ and pairs $(k, t) \in \mathcal{T}$. Likewise, lower bounds on $g_{it}(k)$ are expressed as

$$g_{it}(k) \geq g_{i(t-1)}(k) - R_i^{dn} u_{it} - R_i^{sd}(u_{i(t-1)} - u_{it}) - g_i^{\max}(1 - u_{i(t-1)}); \quad t = 2, \ldots, T, \quad (4.39)$$

$$g_{it}(k) \geq g_{i0} - R_i^{dn} u_{it} - R_i^{sd}(u_{i0} - u_{it}) - g_i^{\max}(1 - u_{i0}); \quad t = 1, \quad (4.40)$$

for all hydrothermal generators $i = 1, \ldots, I$ and pairs $(k, t) \in \mathcal{T}$.

Demand-side constraints

The demand-side limits defined in Appendix B have to be modified here to account for the fact that only demand levels associated with the error-free scenario $(d_{mt}(\hat{k}) = \hat{d}_{mt})$ are bounded by elasticity limits. That is, for $m = 1, \ldots, M$ and $t = 1, \ldots, T$

$$d_{mt}^{\min} \leq d_{mt}(\hat{k}) \leq d_{mt}^{\max}. \quad (4.41)$$

4.2.8 Objective function

As already described in previous chapters, the goal of the electricity market-clearing problem here is also to minimize a measure of the expected social cost. However, given the different structure of the uncertainty factors here, it is possible to distinguish between two components of the expected social cost function:

- There are those components that materialize with a probability of one, which are:
  - The scheduling costs of reserve services (up- and down-spinning, both generation- and demand-side), $C_r(\mathbf{r}^{up}, \mathbf{r}^{dn})$; and,

  - The fixed running and startup costs of the hydrothermal generators that depend on the binary unit commitment variables, $C_g(\mathbf{u})$.
- Those components that materialize with a probability $p_t(k)$ during period t and under scenario $\mathcal{S}_k$, which are:
  - The demand-side benefits, $B_d(\mathbf{d}_t(k))$;
  - The generation-side running costs, $C_g(\mathbf{g}_t(k))$;
  - The operating costs of the energy storage installations, $C_s(z_t^c(k), z_t^d(k))$; and,
  - The costs of involuntary load shedding, $\mathbf{v}_t^T \mathbf{l}_t(k)\Delta$.

The reason why here there are components of the expected social cost function that are assigned a probability of one is that their associated sets of decisions variables (*i.e.* unit commitment and reserve scheduling decisions) have to be taken prior to the revelation of the uncertainty. On the other hand, the components of the expected social cost function which are assigned probabilities $p_t(k)$ are those which only materialize once the uncertainty is revealed. These components measure the expected social cost associated with the reserve deployment patterns needed to keep the power system balanced during the full length of the scheduling horizon and for all the considered scenarios.

In cases where there are some energy storage installations connected to the grid, an extra term may be added to the expected social cost function to weigh in the economic value of energy stored at the end of the scheduling horizon.

In the expected social cost function, we do not assign an operating cost component associated with the WP generators. It is probably not realistic at the present moment for WP generators to submit nonzero running costs offers given their relative incapacity to regulate effectively their output. Revisions of the current "must-take" operational regime[10] for WP will be warranted only when it is possible to properly control the output of wind turbines. Here, we do assume that this regime still applies. Clearly, the development of techniques aimed at the formulation of effective market offering strategies for WP generators warrants further investigation, which is, however, outside of the scope of this dissertation. As mentioned in the Introduction of the current chapter, the development of these techniques are tightly coupled with the development of dependable wind forecasting techniques.

[10] Under this regime, grid operators must take in the WP generation as it is produced in real time regardless of its level and variations. Thus, here the grid operators balance their respective systems using extra amounts of secondary and tertiary reserves. Moreover, the WP generators are remunerated at a fixed and regulated rate (often subsidized), which does not necessarily reflect the market value of electricity [143, 145, 146, 159]. These extra costs are socialized and then levied through an uplift charge.

The expected social cost objective function includes an operating cost term associated with the energy storage installations. Besides fixed capital as well as relatively constant operations and maintenance expenses, the energy storage installations do not have significant variable cost components. Of course, a higher number and faster transitions between charging and discharging cycles should increase the wear and tear of the storage devices, which, as a result, could increase the maintenance costs [177,178]. So here the variable cost of operation could be assumed to be proportional to the charging and discharging rates $[z_t^c(k),\, z_t^d(k)]$.

The lost opportunities caused by the power losses constitute another noteworthy operating cost of energy storage installations. Consider the example where one megawatt-hour is bought at λ_t^E dollars per megawatt-hour and is stored with efficiency η^c and is later released during period t' at an efficiency of η^d when the price of electricity is $\lambda_{t'}^E > \lambda_t^E$ dollars per megawatt-hour. This means that the storage operator incurs a marginal opportunity cost (marginal profit loss) equal to $(1 - \eta^c\eta^d) \times (\lambda_{t'}^E - \lambda_t^E)$ dollars per megawatt-hour.

It remains, however, that the formulation of rigorous operation costing models for energy storage installations—which are probably very technology-dependent—should be the subject of further investigation.

Mathematically, the objective of the electricity market-clearing problem with net load uncertainty boils down to

$$\begin{aligned}\min C_g(\mathbf{u}) + C_r(\mathbf{r}^{up}, \mathbf{r}^{dn}) - \gamma\hat{e}_T - \sum_{(k,t)\in\mathcal{T}} p_t(k)[B_d(\mathbf{d}_t(k)) - C_g(\mathbf{g}_t(k))\\ - \mathbf{v}_t^T\mathbf{l}_t(k)\Delta - C_s(z_t^c(k), z_t^d(k))],\quad (4.42)\end{aligned}$$

where the term $-\gamma\hat{e}_T$ corresponds to the economic value of the expected energy stored at the end of the scheduling horizon.

4.2.9 Incorporating hydrothermal generation contingencies

The analysis of the electricity market-clearing formulation with demand and WP generation uncertainty would be incomplete without the inclusion of a discussion of the impacts of equipment contingencies (*i.e.* hydrothermal generator and centralized energy storage installations outages). From the point of view of the mathematical programming formulation, modeling generator contingencies is straightforward as it boils down to the mere generation of extra scenarios. The scenarios' probabilities are found by calculating the convolution of the (assumed independent) probability distributions of the net load forecast error with those of the generation contingencies [46].

From a computational point of view, however, the addition of hydrothermal generator contingencies would render the solution process of the market-clearing problem much more challenging. In fact, the addition of a single generator contingency, considering its possible times of failure, would multiply the number of scenarios by the number of periods of the scheduling horizon. We must recall that with each extra scenario comes the corresponding extra variables and constraints. As a result, realistically-sized problems may be very hard to handle with current computing tools.

Yet, some simplifications could be considered. One possibility would be to have a hybrid solution whereby a deterministic reserve criterion (*e.g.* $N-1$ criterion or schedule some proportion of the demand) that covers the hydrothermal generation contingencies and have the probabilistic method developed here to take care of scheduling reserves for the WP and demand uncertainty. A second solution path would make use of the scenario reduction techniques mentioned before [105, 121–123, 175]. Finally, not to be neglected, the decomposition techniques evoked previously [31,82,85,112,115,116,118–120] are promising because they can exploit the intrinsic decomposable structure of the problem—whereby each scenario is optimized individually under the command of a master coordinating problem. These aspects are outside of the scope of the current dissertation, but should be subject of extensive future research and development efforts.

4.3 Case Studies

This section outlines the results of case studies of the electricity market-clearing with stochastic security under demand and WP generation uncertainty. The case studies aim at demonstrating that:

- When planning operations under uncertain load and wind forecasts, voluntary and/or involuntary load shedding can be valuable scheduling options for the system operator;
- Under stochastic market-clearing, the acceptable level of wind power penetration can be more important than under operating schemes based on deterministic security criteria founded on worst case scenario situations; and,
- Energy storage installations can bring about improved operational flexibility and

economic savings.

These case studies are based on System A described in Appendix E.1 with the following modifications:

- The demand-side spinning reserve services offers are worth \$50 per megawatt-hour (in lieu of \$20 per megawatt-hour);
- The transmission network is ignored; and,
- Non-spinning reserve services are not considered.

4.3.1 Base case

No wind power without demand uncertainty

Under a perfect demand prediction—where the prediction follows the demand schedule in Table E.1—and no installed wind power capacity, the minimum expected social cost of operation is \$6300.00. Obviously, under this uncertainty-free regime of operation no spinning reserve services are scheduled, while the associated expected marginal costs of security (λ_t^S) are nil for all time periods $t = 1, \ldots, 4$.

The corresponding generation schedule is found in Table 4.1, and the expected marginal costs of energy (λ_t^E) are reported in Table 4.2.

Table 4.1 Uncertainty-free generation schedule

		Time t (h)			
		1	2	3	4
g_{1t}	(MW)	0.0	30.0	60.0	0.0
g_{2t}	(MW)	0.0	0.0	0.0	0.0
g_{3t}	(MW)	30.0	50.0	50.0	40.0

Table 4.2 Uncertainty-free marginal social costs of energy

		Time t (h)			
		1	2	3	4
λ_t^E	(\$/MWh)	20.00	30.00	30.00	20.00

No wind power with demand uncertainty

Next, we consider the same scheduling problem as above, but this time with demand uncertainty. Here the standard deviation of the demand forecast error is assumed to equal 2% of the hourly demand prediction found in Table E.1. The demand prediction error probability distribution is approximated by a discrete distribution made up of seven one-standard deviation-wide slices as shown in Fig. 4.1. This slicing arrangement gives rise to a 2401-scenario scenario tree for the four hour-long scheduling horizon.

The minimum expected cost under stochastic market-clearing is \$6478.52 (a 2.83% increase over the uncertainty-free case reported above). The expected cost breakdown for schedules based on the deterministic—which does not allow the use of involuntary load shedding—and the stochastic market-clearing are in shown in Table 4.3. By inspection, the components corresponding to the error-free scenario (that include generator startup costs) and the reserve deployment operations represent the bulk of the total expected cost under both deterministic and stochastic formulations. As one would expect, in light of the conclusions of Chapter 2, the stochastic scheduling method does bring in expected savings [value of the stochastic solution (VSS)] of \$2.69, which correspond to 0.04% of the expected cost under the deterministic security criterion. We note that this improvement is quite small. This is explained by the small demand forecast uncertainty associated with this particular case.

The generation schedules corresponding to the error-free scenario, for both the deterministic and stochastic cases, follow the one found in the uncertainty-free case shown in Table 4.1. The generation- and demand-side spinning reserve schedules corresponding to the stochastic case are found in Table 4.4. In addition, the last row of Table 4.4 lists the amount of expected load not served during each of the four hours of the scheduling horizon, where the ELNS is calculated as

$$\hat{l}_t = \sum_{k:(k,t)\in\mathcal{T}} p_t(k) l_t(k) \Delta. \tag{4.43}$$

During periods $t = 1$ and $t = 4$ the up-going spinning reserve provided is not sufficient to cover the most extreme cases of high demand under-prediction—both corresponding to scenarios where the forecast is off by three load forecast error standard deviations. On the other hand, the down-going spinning reserve has to cover the whole range of under-

Table 4.3 Comparison of expected social costs under demand uncertainty—deterministic versus stochastic

	Total	Reserve scheduling	Error-free scenario[a]	Reserve deployment	Loss-of-load
Deterministic ($)	6481.20	181.20	882.77	5417.23	0.00
Stochastic ($)	6478.52	170.00	882.77	5417.05	8.69
Difference ($)	2.69	11.20	0.00	0.18	−8.69

[a]Includes startup costs.

Table 4.4 Reserves and ELNS under demand uncertainty

		Time t (h)			
		1	2	3	4
r_{1t}^{up}	(MW)	0.0	4.8	6.6	0.0
r_{1t}^{dn}	(MW)	0.0	4.8	6.6	0.0
r_{2t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
r_{3t}^{up}	(MW)	1.2	0.0	0.0	1.6
r_{3t}^{dn}	(MW)	1.8	0.0	0.0	2.4
r_{dt}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{dt}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_t$	(kWh)	3.7	0.0	0.0	5.0

prediction errors since "negative" load shedding (generation spillage) is not modeled here; that explains why the values of r_{3t}^{up} are less than those of r_{3t}^{dn} during $t = 1$ and 4.

Finally here, Table 4.5 lists the expected marginal costs of energy and security corresponding to the stochastic schedule. These marginal costs were calculated from the principles outlined in Appendix G and extended in Chapter 3, taking the error-free scenario as the pre-contingency state. Table 4.5 illustrates well that the expected marginal costs of security represent an important proportion of the expected marginal costs of energy. This does indeed reflect the fact that even small forecasting errors have significant impact on the marginal costs of energy and security.

Table 4.5 Expected marginal social costs of energy and security under demand uncertainty

		Time t (h)			
		1	2	3	4
λ_t^E	(\$/MWh)	20.00	30.00	30.00	20.00
λ_t^S	(\$/MWh)	12.34	25.60	28.32	19.57

4.3.2 Wind penetration level and wind prediction uncertainty

In this section, we demonstrate how the stochastic market-clearing formulation can accommodate better the WP generation and the demand forecast errors than deterministic-based security-constrained market-clearing. As the WP generation penetration[11] level is increased, for some arbitrary WP and demand predictions, the stochastic market-clearing is solved and its outcomes are analyzed against the outcomes of a corresponding deterministic market-clearing formulation. Here the deterministic market-clearing formulation is one which schedules reserves so as to counter all possible realizations of the net load forecast error scenarios without involuntary load shedding or wind energy spillage. As done in Chapter 2 and above, under demand uncertainty only, comparisons between the deterministic and the stochastic-based market-clearing solutions are made by examining the differences between the respective expected values of a number of quantities.

[11] WP penetration is defined as the ratio of the WP installed generation capacity to the total hydrothermal capacity, which is 250 MW here.

Here, the WP forecast error model is taken from Fabbri *et al.* [147], under which the standard deviation of a WP forecast error is estimated from a function of the normalized predicted power. Assuming that the WP prediction was made 24 hours prior to the first hour of the schedule and for a region size of 140 kilometers, the standard deviation of the WP forecast error is approximated by

$$\sigma_{wt} = 0.02 + 0.2\,\hat{w}_t, \tag{4.44}$$

where σ_{wt} and $\hat{w}_t$ are in per unit of the installed wind capacity for $t = 1, \ldots, 4$. The per-unitized WP generation forecast $\hat{w}_t$ that will be used is found in Table 4.6.

Table 4.6 Hourly wind power generation forecast

	Time t (h)			
	1	2	3	4
$\hat{w}_t$ (p.u.)	0.55	0.35	0.10	0.25

Fig. 4.3 shows the evolution of the expected social cost of the schedule for both the deterministic and stochastic market-clearing approaches as the WP generation penetration level is increased. The expected cost under stochastic market-clearing undergoes a steady decrease as the amount of WP generation increases. This contrasts with the deterministic-based cost which decreases until the 8% penetration mark where it starts increasing until no feasible schedules exist, when the penetration level reaches 10%. The observed behavior of the deterministic-based expected scheduling cost reflects that at one point the need for reserves to cover all uncertainty scenarios overwhelms the expected savings brought about by the increasing level of free, but uncertain, WP generation. It is also interesting to note here that the "free" wind power compensates for its extra reserve needs starting at a fairly low penetration level. It only takes a penetration level of about 3% for the expected costs (stochastic and deterministic) to be less than the schedule cost under the base case with a perfect demand forecast and no wind, which we found in Section 4.3.1 ($6300).

The size of the gap between the two curves represents the value of the stochastic solution (VSS), which is plotted in per unit of the deterministic expected cost in Fig. 4.4. We observe the slow increase in the VSS until the 8% penetration level. The faster increase in between 8 and 9% is explained by the expensive demand-side reserves (at $50 per megawatt-hour)

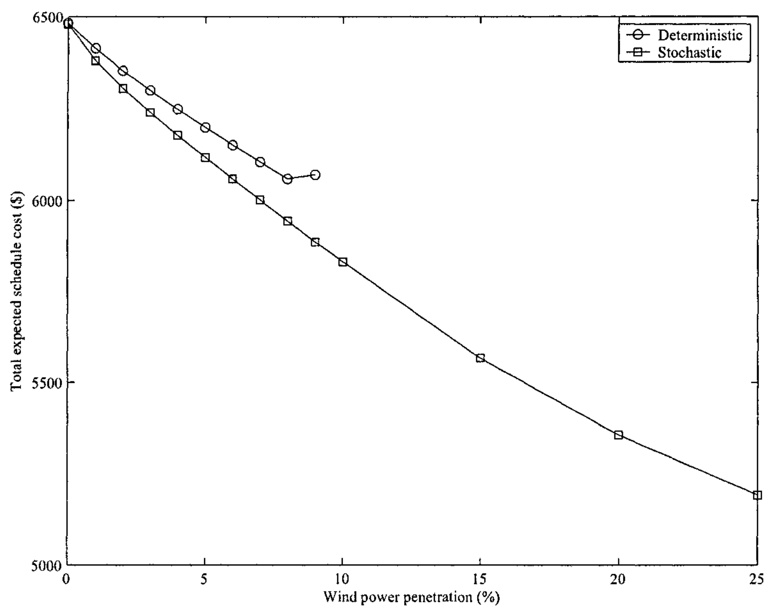

Fig. 4.3 Total expected social costs of scheduling as a function of the WP penetration

needed to balance power for all the uncertainty scenarios under the deterministic case—even those with very low probabilities.

The behavior of the expected costs and the VSS can be further investigated by inspecting Fig. 4.5 that shows the progression of the reserve costs as the WP penetration varies. There is nothing more to say about the case of the deterministic-based market-clearing; it is clear that the reserve costs increase sharply with the level of WP penetration. On the other hand, the cost of reserves under stochastic market-clearing follows a different pattern: (i) between 0 and 2% of WP penetration, the cost of reserves decreases because, on average, it is less expensive to spill wind and involuntarily shed load than schedule more reserves given the small size of the WP capacity; (ii) between 2 and 15% of WP penetration, more reserves are required since the cost of load shedding and the opportunity cost of spilling wind increase faster than the cost of reserves; and, (iii) for 15% of WP onward, the cost of reserves is steady as more reserves cannot further decrease the global scheduling costs. These phenomena are reflected in the amounts of involuntary load shedding and wind energy spillage shown in Fig. 4.6 and 4.7 respectively. The faster increase in the wind energy spillage occurring for WP penetration levels above 15% is reflected in the flat part of the cost of reserves curve (Fig. 4.5) as, under these penetration levels, wind energy spillage,

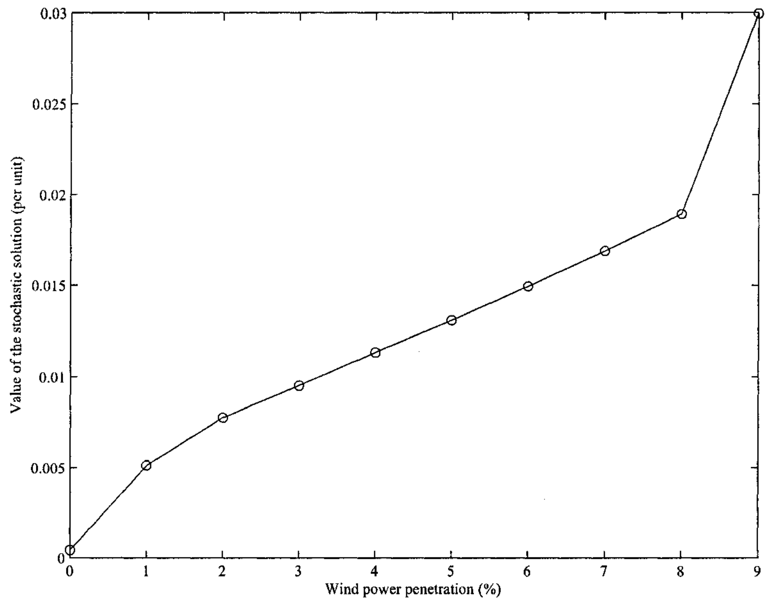

Fig. 4.4 Value of the stochastic solution as a function of the WP penetration

which is free, is used in lieu of down-spinning reserve.

One of the peculiarities of the demand and wind forecasts in this case study is the occurrence of a low demand period during $t = 1$, which is also a high wind period. This has serious implications as can be seen in Table 4.7 for the particular case when the WP penetration level equals 15%. During the first period, generator 3 is constrained by its minimum output level (10 megawatts), impeding down-going regulation actions. As a result, plenty of wind energy is spilled—an average of 2306.3 kWh during this period. This constraint on down-going actions is reflected in the lower value of the expected marginal costs of energy and security during that hour shown in Table 4.8 (with respect to those in Table 4.5 where only demand uncertainty is affecting the schedule). These lower marginal expected costs are clear economic signals indicating that, by increasing their load during that period, consumers could benefit from the plentiful supply of free wind.

4.3.3 Energy storage

In light of the expected amounts of spilled wind energy found in the above example, it is of interest to investigate how a large-scale (megawatt-hour-range) energy storage facility could lower the overall expected costs and make better use of the installed wind capacity.

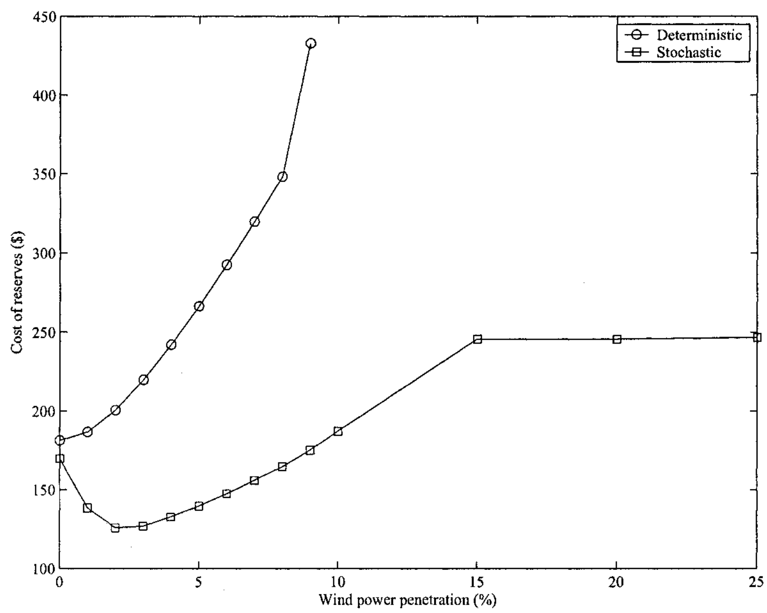

Fig. 4.5 Cost of reserves as a function of the WP penetration

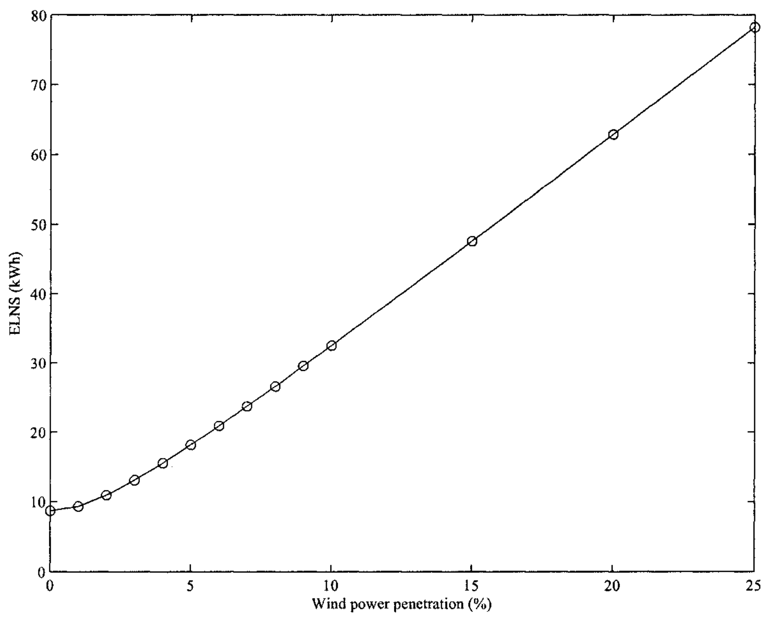

Fig. 4.6 ELNS as a function of the WP penetration

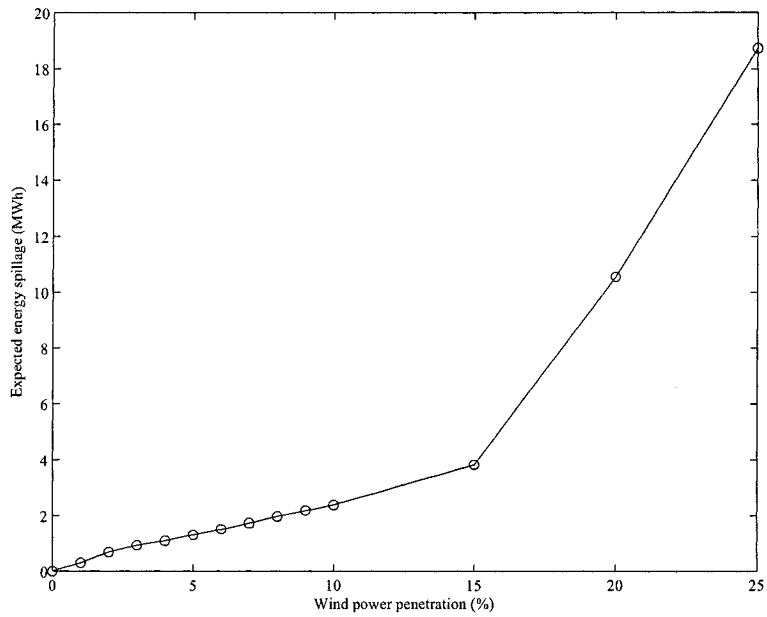

Fig. 4.7 Expected wind energy spilled as a function of the wind power penetration

Table 4.7 Error-free generation, reserves, ELNS and expected wind energy spillage for 15% WP penetration

		Time t (h)			
		1	2	3	4
$g_{1t}(\hat{k})$	(MW)	0.0	16.9	56.3	0.0
r_{1t}^{up}	(MW)	0.0	11.2	8.0	0.0
r_{1t}^{dn}	(MW)	0.0	3.7	2.7	0.0
$g_{2t}(\hat{k})$	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$g_{3t}(\hat{k})$	(MW)	10.0	50.0	50.0	30.6
r_{3t}^{up}	(MW)	9.2	0.0	0.0	5.5
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
r_{dt}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{dt}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_t$	(kWh)	30.5	0.0	0.0	17.0
$\hat{s}_t$	(kWh)	2306.3	272.7	194.4	1047.1

Table 4.8 Expected marginal social costs of energy and security for 15% WP penetration

		Time t (h)			
		1	2	3	4
λ_t^E	(\$/MWh)	14.17	30.00	30.00	20.00
λ_t^S	(\$/MWh)	14.17	25.60	28.32	20.00

Here the goal is to show how the energy storage capacity $e^{\max}$ (in megawatt-hours) relative to the installed WP generation capacity (in megawatts) affects: (i) the expected scheduling cost; (ii) the cost of reserves; (iii) the expected load not served; and, (iv) the expected value of the spilled wind energy.

For conciseness here, we consider only the effects of lossless energy storage applied when the WP generation penetration equals 10% (25 megawatts) in System A under stochastic market-clearing. Moreover, we assume that:

- The WP generation and demand forecasts and their corresponding uncertainties are as in Section 4.3.2;
- The operating costs of the energy storage facility are neglected;
- The maximum rate of charge and discharge $z^{\max}$ is set equal to $e^{\max}/\Delta$, where $\Delta =$ 1 hour;
- The storage ramping rate $\bar{z}^{\max}$ is assumed to be larger than twice $z^{\max}$, which implies that the storage facility can switch from fully charging to fully discharging—and vice versa—within a single time period;
- The initial energy stored e_0 is set equal to half of the storage capacity, $e_0 = e^{\max}/2$;
- There is no lower bound on the expected value of the energy stored in the last time period. Instead, we value the energy stored in the last period at the marginal rate γ, which is set equal to \$10 per megawatt-hour.

Fig. 4.8 shows the near-linear decrease in the expected value of the scheduling cost with the increasing proportion of storage capacity to the installed WP generation. This expected cost curve is clear of the term measuring the expected value of the energy stored at the end of the scheduling horizon, $\gamma\hat{e}_T$, appearing in (4.42). It can be seen that under the current level of WP penetration (10%), the incremental reduction in the expected scheduling cost is roughly \$20 per megawatt-hour of added storage capacity. It should be noted as well that the bulk of the expected cost decrease seen here is due to the steady reductions in the reserve deployment costs with the increase in the storage to WP capacity ratio. The reserve deployment component is reduced because the storage installations essentially are replacing the use of classical reserve services.

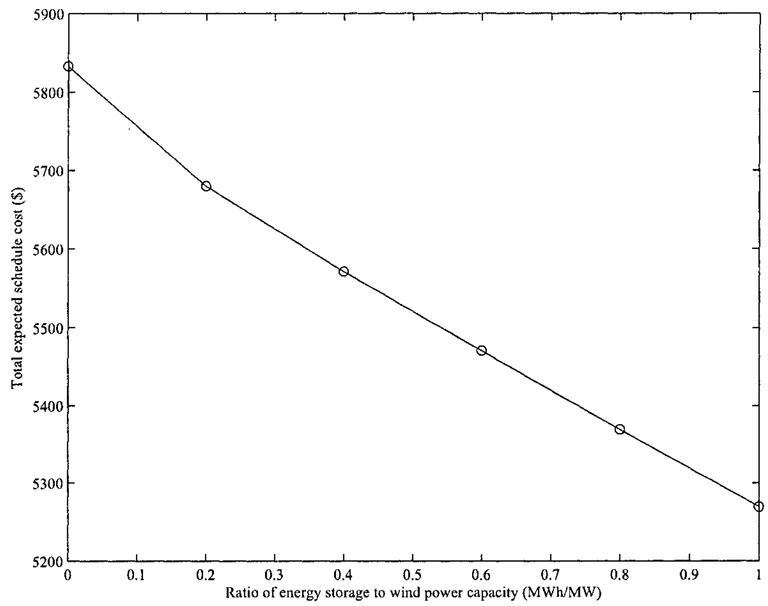

Fig. 4.8 Expected social cost of scheduling as a function of the relative energy storage capacity at 10% WP penetration

An allied observation is that, as seen in Fig. 4.9, the cost of reserves scheduling decreases with the increase in the proportion of the installed storage to the WP generation capacity. This also indicates that the energy storage is clearly a substitute for classical reserve services here. However, the decrease becomes quasi nil as the storage to WP capacity ratio reaches 0.6 MWh/MW. This change in behavior is explained by the fact that when the storage capacity is large enough, it is no longer used just to buffer the variations in the net load. More fundamentally here, the reason why the cost of reserve scheduling is no longer decreasing comes from the economic arbitrage opportunities offered by the storage installations.

For instance, when the storage to WP generation capacity ratio is set to 1.0 MWh/MW, during period $t = 1$ under the error-free scenario, generator 3 is scheduled to generate power (at $20 per megawatt-hour) much over the predicted net load of 11.25 megawatts, as can be seen in Table 4.9. In this case, the surplus generation is stored to be released later

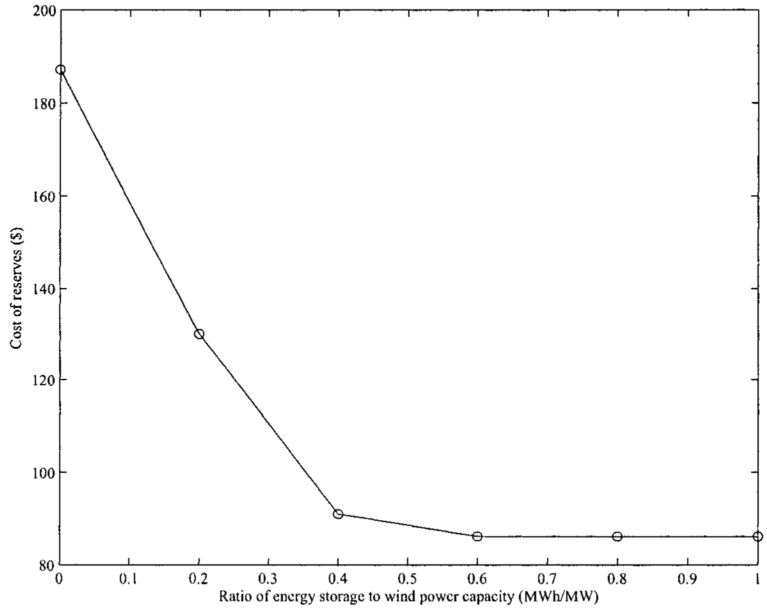

Fig. 4.9 Costs of reserves as a function of the relative energy storage capacity at 10% WP penetration

in periods 2 and 3 during which the predicted net load is higher and the more expensive generator 1 (generating at $30 per megawatt-hour) is needed to balance power.

In a nutshell, with sufficiently large storage, the reserve scheduling, load shedding and wind spillage savings are no longer the dominant factors in the market-clearing decision process. This is reflected further in the behavior of the ELNS and the expected value of spilled wind energy as shown in Fig. 4.10 and 4.11 respectively.

Not to be neglected here, there are other key parameters which could have significant effects on the behavior of the market-clearing problem in the presence of energy storage. Clearly, ramping and charging/discharging cycling restrictions of the energy storage facilities are the ones that could affect greatly the performance of the market as a whole.

4.3.4 Computational complexity

The dimensions of the simple market-clearing problems just studied are not trivial; they are reported in Table 4.10. As mentioned before, the large dimensions are a consequence of the fact that no scenario reduction techniques were applied to the formulation. Moreover, the number of net load forecast error probability distribution slices (seven) is a factor directly influencing the size of the problems. Obviously, fewer slices would reduce the problem size

Table 4.9 Error-free generation, reserves, ELNS, expected wind energy spillage and storage for 10% WP penetration with 1.0 MWh/MW storage to WP capacity ratio

		Time t (h)			
		1	2	3	4
$g_{1t}(\hat{k})$	(MW)	0.0	13.0	40.8	0.0
r_{1t}^{up}	(MW)	0.0	0.0	13.3	0.0
r_{1t}^{dn}	(MW)	0.0	0.0	0.8	0.0
$g_{2t}(\hat{k})$	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{2t}^{dn}	(MW)	0.0	0.0	0.0	0.0
$g_{3t}(\hat{k})$	(MW)	28.7	50.0	50.0	35.7
r_{3t}^{up}	(MW)	0.0	0.0	0.0	1.9
r_{3t}^{dn}	(MW)	0.0	0.0	0.0	0.0
r_{dt}^{up}	(MW)	0.0	0.0	0.0	0.0
r_{dt}^{dn}	(MW)	0.0	0.0	0.0	0.0
$\hat{l}_t$	(kWh)	0.0	0.0	7.6	8.8
$\hat{s}_t$	(kWh)	1261.0	0.0	0.0	0.0
$\hat{e}_t$	(MWh)	23.7	15.5	0.9	2.9

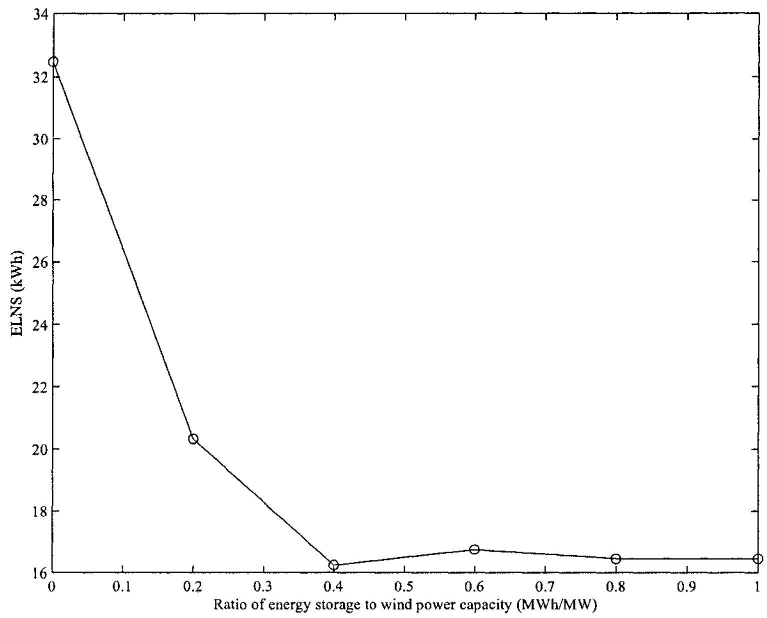

Fig. 4.10 ELNS as a function of the relative energy storage capacity at 10% WP penetration

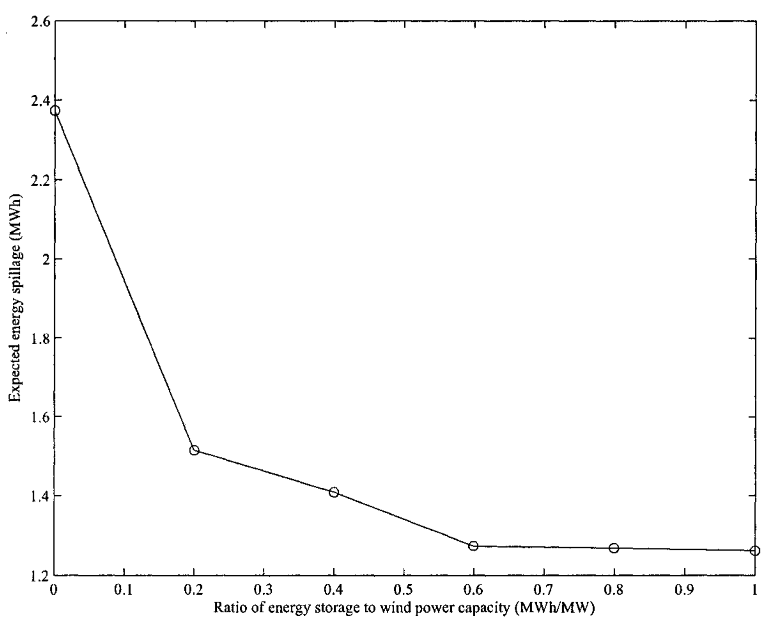

Fig. 4.11 Expected wind energy spilled as a function of the energy storage capacity at 10% WP penetration

but at the expense of modeling accuracy.

Table 4.10 Dimensions of stochastic market-clearing problems under demand and WP uncertainty—with and without energy storage

	Without storage	With storage
Constraints	93 660	107 660
Variables	33 657	39 257
Binaries	12	12

The market-clearing problems were solved with *Paco* using version 9.0.2 of CPLEX (see Appendix F). As seen in Chapter 2, the mixed-integer linear programming solver pre-processing engine is generally capable of reducing significantly the size of the market-clearing problems. For the above cases, the reduced problems have the dimensions given in Table 4.11. For example, the solution times were of 39.0 seconds and 89.5 seconds for the 10% WP generation penetration cases respectively without storage and with 0.4 MWh/MW of lossless storage. Besides their larger number of variables, cases with storage are longer to solve because they involve more of the complicating inter-temporal constraints of the form found in (4.15), (4.16), (4.18) or (4.28).

Table 4.11 Dimensions of stochastic market-clearing problems under demand and WP uncertainty—with and without energy storage—with CPLEX pre-processing applied

	Without storage	With storage
Constraints	39 209	39 209
Variables	14 000	16 800
Binaries	12	12

Again, reiterating what has already been mentioned regarding the computational complexity of the proposed formulations, any practical implementation of the proposed formulations would require further investigation of scenario reduction techniques applied specifically to the current problem. Moreover, in conjunction with research on scenario reduction techniques, the application of decomposition methods should equally be the subject of future research efforts.

4.4 Summary

Current electricity market-clearing schemes cannot fully integrate the most essential feature of demand and non-dispatchable generation technologies like solar and wind power. This limitation of market-clearing schemes is becoming an issue for grid operators as there is more and more public and political pressure to increase the penetration of renewable generation technologies, which depend on ever-evolving weather conditions.

Using the stochastic security framework developed earlier in the dissertation, this chapter proposed an electricity market-clearing formulation that can account explicitly for those uncertainties. Using wind power generation as a typical non-dispatchable generation technology, we formulated a stochastic market-clearing problem wherein the uncertainty in the demand and the wind power are approximated by a joint discrete probability distribution. In addition, we extended the basic market-clearing formulation to permit the participation in the market of large-scale energy storage installations.

Case studies showed how the proposed formulation can effectively reduce expected operating costs and can extend the feasible wind power generation penetration level in comparison to "worst-case" scenarios deterministic operational planning techniques. In addition, we showed that the addition of large-scale energy storage facilities can improve the economic efficiency of the market.

Chapter 5

Umbrella Contingencies in Security-Constrained Market-Clearing

Cover my defenceless head
With the shadow of thy wing.

In Temptation
Charles Wesley, 1708–1788

5.1 Introduction

We have seen throughout the previous chapters of this dissertation that integrating security constraints in an electricity market-clearing formulation (be it deterministic or stochastic) poses important computational challenges. On top of the underlying computational explosion, lies the tight time limits faced by grid and market operators as they resolve day-ahead and hour-ahead electricity scheduling problems. As a result, research efforts aiming at reducing the dimensions of the security-constrained market-clearing problems are paramount to render these tools more practical for the industry.

We enumerated before a number of possible techniques which may be used to improve the computational tractability of such tools. Techniques based on decomposition methods [27, 31, 82, 85, 112, 115, 116, 118–120] are applicable to both deterministic and stochastic versions of security-constrained market-clearing formulations. Only applicable to stochastic instances of the security-constrained market-clearing problem, the use and development of scenario reduction techniques [121–123] also represent significant steps in the desired direction.

In the more general context of power system operation, past and current practices approached the large number of contingencies to be analyzed through the process of contingency ranking [179–184]. These ranking procedures are usually based on simplified load flow or transient stability analyses for each of the credible contingencies relative to a given operating point previously found by an economic dispatch or a unit commitment. Under such a ranking paradigm, only those highly ranked contingencies are analyzed in detail, from which any necessary preventive security control actions are then defined and implemented [41, 185, 186].

Contingency ranking is closely tied to the notion of umbrella contingencies. We say that a contingency k_0 is an umbrella contingency of the set of contingencies $\{k_1, \ldots, k_n\}$ if the condition that the system is secure with respect to k_0 only implies that the system is also secure with respect to $\{k_1, \ldots, k_n\}$. Thus, it is readily seen that evaluating the system security with respect to k_0 only is equivalent to evaluating the system security with respect to the set $\{k_0, \ldots, k_n\}$. We can say then that the set of umbrella contingencies is the reduced collection of contingencies comprising only those sufficient to cover the entire set of credible contingencies. The notion of umbrella contingencies is therefore much more powerful than that of a ranked list of contingencies since the latter does not eliminate a contingency from further consideration. Umbrella contingencies are used extensively by system operators to limit the number of credible contingencies to be analyzed. The basis for the identification of umbrella contingencies is based mostly on empirical evidence and experience [186], although some related set-theoretic results based on security regions have been developed during the 1970's and 1980's [187–190].

In this chapter, we investigate a rule for the identification of umbrella contingencies for security-constrained electricity market-clearing problems based on optimal power flow [191]. Specifically, the goal here is to predict, in terms of the anticipated system conditions, reduced numbers of equivalent credible umbrella contingencies and variables for which the resolution of the corresponding relaxed security-constrained optimal power flow results in the same or nearly the same market-clearing schedule as with the full set of contingencies. It is readily seen that having to resolve a security-constrained market-clearing problem constrained only by its set of umbrella contingencies should be much simpler and computationally cheaper than having to resolve a similar problem, but, this time, constrained by all credible contingencies.

In this chapter, we propose that the members of the set of umbrella contingencies can be systematically found from the Lagrange multipliers of the underlying full security-constrained power flow (SCOPF) problem. In the deterministic version of the market-clearing problem, we show how to identify a set of umbrella contingencies that yields precisely the same market-clearing solution as that of the full SCOPF. On the other hand, in the stochastic case, a smaller set of umbrella contingencies can be obtained such that the sensitivity of the optimum solution to the neglected contingencies is less than some pre-determined threshold.

For the deterministic case, we present a numerical study illustrating how the system demand can affect the membership of the umbrella set. In the stochastic case, we examine the heuristic threshold rule defining membership in the umbrella set as well as its impact on the market-clearing solution.

5.2 Umbrella Contingencies

5.2.1 Security-constrained market-clearing

Consider the security-constrained optimal power flow problem optimizing $W(\mathbf{u}, \mathbf{x})$, which represents a measure of the social cost over the pre- and post-contingency operating states, where the latter are defined by a set of credible contingencies $\mathcal{K}$

$$\min_{\mathbf{u},\mathbf{x}} W(\mathbf{u}, \mathbf{x}), \tag{5.1}$$

subject to the nodal pre-contingency power balance conditions

$$\mathbf{H}(\mathbf{u}(0), \mathbf{x}(0), 0) = \mathbf{0} \quad (\boldsymbol{\mu}(0)), \tag{5.2}$$

the nodal power balance conditions for each of the contingencies $k \in \mathcal{K}$

$$\mathbf{H}(\mathbf{u}(k), \mathbf{x}(k), k) = \mathbf{0} \quad (\boldsymbol{\mu}(k)), \tag{5.3}$$

and all inequalities applying to all pre- and post-contingency variables

$$\mathbf{G}(\mathbf{u}, \mathbf{x}) \geq \mathbf{0} \quad (\boldsymbol{\sigma}). \tag{5.4}$$

Here the vector $\mathbf{u}$ represents the entire set of discrete variables of the SCOPF such as on/off generator status and transformer tap settings, while $\mathbf{x}$ represents the continuous generation, demand and reserve levels, bus voltages and, possibly, load shedding. We point out that the vectors $\mathbf{u}$ and $\mathbf{x}$ include both pre- and post-contingency variables.

Associated with their respective constraints, (5.2), (5.3) and (5.4), we define the Lagrange multiplier vectors $\boldsymbol{\mu}(0)$, $\boldsymbol{\mu}(k); \forall k \in \mathcal{K}$, and $\boldsymbol{\sigma}$.

In the case where the market-clearing problem is based on a *deterministic* SCOPF (like in Section 2.2), the objective function $W(\mathbf{u}, \mathbf{x})$ ignores the probability of occurrence of the contingencies. Moreover, it does not optimize the associated loss of welfare due to reserve deployment following the occurrence of any credible contingency as in [64].

This is unlike the *stochastic* case where the objective function is the expected value of the social cost that considers the probabilities of occurrence of the credible contingencies and the expected associated extra incurred costs due to reserve deployment (as proposed in Section 2.4). For example, if the contingencies $k \in \mathcal{K}$ have probabilities $p(k)$ and the pre-contingency state has a probability $p(0)$, the objective function of the stochastic SCOPF is written as

$$W(\mathbf{u}, \mathbf{x}) = p(0)W(\mathbf{u}(0), \mathbf{x}(0), 0) + \sum_{k \in \mathcal{K}} p(k)W(\mathbf{u}(k), \mathbf{x}(k), k), \tag{5.5}$$

where $W(\cdot, 0)$ and $W(\cdot, k)$ express respectively the pre- and post-contingency social cost functions.

Unlike its deterministic counterpart, by accounting for the post-contingency expected social welfare, the stochastic SCOPF is well suited to balance the relative benefits and costs of reserve scheduling and deployment versus those of load shedding, as seen in Chapter 2.

5.2.2 Identifying umbrella contingencies

Let us consider a subset of the set of credible contingencies, $\mathcal{U} \subseteq \mathcal{K}$. We can define a relaxation of the full SCOPF, (5.1)–(5.4), replacing (5.3) by

$$\mathbf{H}(\mathbf{u}(k), \mathbf{x}(k), k) = \mathbf{0}; \quad \forall k \in \mathcal{U}. \tag{5.6}$$

Let us denote the solution to the full SCOPF by the triplet $(W, \mathbf{u}, \mathbf{x}) = (W^*, \mathbf{u}^*, \mathbf{x}^*)$ and that of the relaxed SCOPF [defined by (5.1), (5.2), (5.4) and (5.6)] by the triplet

$(W, \mathbf{u}, \mathbf{x}) = (W^\dagger, \mathbf{u}^\dagger, \mathbf{x}^\dagger)$. We should note here that the solution vectors $(\mathbf{u}^\dagger, \mathbf{x}^\dagger)$ of the relaxed SCOPF problem are of lower dimension than their full SCOPF counterparts since the relaxed SCOPF does not need to optimize over the omitted contingencies and their corresponding variables.

Consider the following alternate definition of the notion of the set of umbrella contingencies, which is a reinterpretation of the one previously made in the Introduction of the present chapter.

Definition 5.1 (Set of umbrella contingencies). The subset $\mathcal{U}$ is a *set of umbrella contingencies* of the full SCOPF if the optimal value of the relaxed SCOPF, $W^\dagger$, is the same or very close to that of the full SCOPF, W^*.

In other words, if, by ignoring a contingency k, the optimal social cost of the SCOPF changes significantly—meaning that this contingency is determinant in setting the optimal solution of the SCOPF—, then this contingency k should be part of the set of umbrella contingencies $\mathcal{U}$. On the other hand, if the corresponding optimal cost of the relaxed SCOPF, found without that contingency, is the same or very close to that of the SCOPF including that contingency, then $k \notin \mathcal{U}$.

Using the above definition as our starting point, we propose next a technique to identify the set of umbrella contingencies of a SCOPF problem formulation from its $\boldsymbol{\mu}(k)$ vectors, the Lagrange multipliers of the post-contingency power balance relations.

Proposition 5.1 (Identification rule for umbrella contingencies). *The elements of the set of umbrella contingencies of the full SCOPF correspond to those contingencies whose associated Lagrange multiplier vectors satisfy $\|\boldsymbol{\mu}(k)\|_p \geq \varepsilon$, for some pre-specified threshold $\varepsilon > 0$.*

The validity of Proposition 5.1 bases itself on the following reasoning. Similar arguments to those found in Chapter 3 and in [192] can be invoked to demonstrate that the sensitivity of the full SCOPF objective function to an infinitesimal perturbation $d\mathbf{S}(k)$ of the right-hand side of the power balance relation under contingency k is

$$\frac{\partial W}{\partial \mathbf{S}(k)} = \boldsymbol{\mu}(k). \tag{5.7}$$

Then, if the sensitivity of the objective function to a perturbation of the power balance relations following contingency k is small, *i.e.* $\|\boldsymbol{\mu}(k)\|_p < \varepsilon$, then the power balance under contingency k can be perturbed without affecting significantly the optimal objective value of the full SCOPF, W^*. This implies that if $\|\boldsymbol{\mu}(k)\|_p < \varepsilon$, then ignoring contingency k in the relaxed SCOPF will likewise not significantly affect the corresponding optimum objective function value ($W^\dagger$) compared to that of the full SCOPF (W^*).

Put differently, for a small enough perturbation of the post-contingency power balance relations associated with contingency k, $d\mathbf{S}(k)$, we have $dW = W^* - W^\dagger = \boldsymbol{\mu}^T(k)\, d\mathbf{S}(k)$ so that

$$\begin{aligned} |dW| &\leq \|\boldsymbol{\mu}(k)\|_p \, \|d\mathbf{S}(k)\|_p \\ &\leq \varepsilon \|d\mathbf{S}(k)\|_p, \end{aligned} \tag{5.8}$$

where we made use of Schwarz's inequality. The above result thus indicates that only those contingencies, whose upper bound on the marginal effect on the optimal SCOPF cost is greater than ε dollars per megawatt-hour, will be admitted in the set of umbrella contingencies. As a result, finding the set of umbrella contingencies simply boils down to identifying those failures that affect most significantly the optimal value of the objective function, while those contingencies having no or little effect on the objective can be disregarded, in accordance with Definition 5.1.

Whereas the above conclusion is theoretically sound when comparing the objective functions of the full and relaxed SCOPFs, additional numerical tests are still needed to assess the validity of the optimal optimization variables obtained with the relaxed SCOPF subject to the set of umbrella contingencies only. The results of the numerical studies, shown in Section 5.4 of the current chapter, suggest that the identification of umbrella contingencies based on the Lagrange multipliers is valid.

5.3 Discussion

The vector norm of the Lagrange multipliers used in Proposition 5.1 could be interpreted as a severity index which can be used to rank the contingencies as already done by power system operators. Here, these severity indices represent some measures of the marginal social costs associated with the different contingencies, which reflect the corresponding marginal social costs of scheduling and deploying reserves. The severity index concept can lead to proposing an alternative to the specification of a cutoff threshold ε, as one could define the set of the umbrella contingencies by retaining only a pre-specified number of the most highly-ranked contingencies.

In addition, the choice of the vector norm type ($p = 1, 2, \infty$, etc.) should have an effect on the selection of the umbrella contingencies. The one and two norms average out the Lagrange multiplier vectors over the entire network, while the infinity norm seeks the extreme values only. For instance, consider a case under which contingency k leads to line congestion that isolates a demand bus m from the rest of the network. Thus, this contingency could cause a large gap between the elements of the vector $\boldsymbol{\mu}(k)$—very high $\mu_n(k)$ for $n = m$ and low $\mu_n(k)$ for $n \neq m$. One can readily see that the ranking rule based on the infinity norm would probably give higher weight to this contingency than under the one or the two norm. Hence, as a general rule we could say here that the one and two norms should favor the contingencies with important network-wide effects, while, on the other hand, the infinity norm should favor the contingencies having important but more localized effects.

It is somewhat obvious that the set of umbrella contingencies for a given SCOPF is strongly dependent on the parameters of the SCOPF (loading, network configuration, unit commitment, etc.). Hence, as the parametrization of the problem changes, the members of the set of umbrella contingencies should also vary. Nevertheless, for most practical situations, the set of umbrella contingencies generally stays constant in a neighborhood of a given parametrization. Moreover, an interesting associated problem here is the determination of contingencies which remain umbrella contingencies independently of the value of a given system parameter of subset of parameters. Identifying these "super-umbrella" contingencies is of clear interest because of their intrinsic insensitivity to parameter uncertainties.

The intended use of the proposed umbrella contingency identification scheme is as an offline market operations planning tool whose role would be to specify a set of umbrella contingencies associated with particular (predicted) operating conditions. For instance, simulations based on predicted operating conditions as well as past post-contingency Lagrange multiplier data could be used to pre-specify a relaxed set of contingencies to constrain an actual market-clearing SCOPF, for which the computational complexity must be kept within reasonable limits. Likewise, the post-contingency Lagrange multiplier data could be of use to system planners when justifying reliability reinforcements. Clearly, an equipment failure having large associated Lagrange multiplier norms on average over some time horizon could indicate that adding a redundant component in parallel could well reduce the economic impact of that failure.

5.4 Case Studies

In this section, we study the proposed umbrella contingency identification rule on a modified version of System A described in Appendix E.1. For simplicity here, we consider only cases without time-dynamic effects, a network model based on the dc load flow, and we neglect unit commitment and thus assume that all three generators are online.

Specifically, the generator data, given in Table E.2, assumes that generator i: (i) produces energy at the rate of g_i megawatts in the range $[\,0, g_i^{\max}\,]$[1] for an incremental cost of a_i dollars per megawatt-hour; (ii) provides up-spinning reserve, r_i^{up}, and down-spinning reserve, r_i^{dn}, at the rates of q_i^{up} and q_i^{dn} dollars per megawatt-hour respectively; and, (iii) has a forced-outage rate given by U_i.

On the load side, we assume that the consumer at bus 3 is inelastic and offers to reduce or increase its consumption in the form of up- (r_d^{up}) or down- (r_d^{dn}) spinning reserve up to 10% of its scheduled consumption at rates $q_d^{up} = \$20$ per megawatt-hour and $q_d^{dn} = \$20$ per megawatt-hour respectively. The inelasticity assumption on the demand-side implies that the objective function of the market-clearing SCOPF minimizes the cost of scheduling generation and reserve services only.

Here, the set of credible contingencies $\mathcal{K}$ includes all single generator and line outages. Contingencies $k = 1, 2, 3$ correspond to the failure of generators 1, 2 and 3 respectively, while contingencies $k = 4, 5, 6$ correspond respectively to the failure of lines 1, 2 and 3.

5.4.1 Deterministic security-constrained market-clearing

This first example examines how Proposition 5.1 works for the deterministic SCOPF. Here, we investigate how the membership of the set of umbrella contingencies is subject to demand variations. We formulate a linear objective function for the market-clearing SCOPF in the following manner

$$\min \mathbf{a}^T \mathbf{g} + (\mathbf{q}^{up})^T \mathbf{r}^{up} + (\mathbf{q}^{dn})^T \mathbf{r}^{dn} + (\mathbf{q}_d^{up})^T \mathbf{r}_d^{up} + (\mathbf{q}_d^{dn})^T \mathbf{r}_d^{dn}. \tag{5.9}$$

[1] Note here that the lower generation limits are assumed to be equal to zero for all three generators, unlike what is indicated in Table E.2.

Table 5.1 reports the Lagrange multipliers of the post-contingency bus power balance equality constraints as a function of the load at bus 3. For each of the load ranges, it is clear that the umbrella contingencies are those for which $\|\boldsymbol{\mu}(k)\|_p > 0$, regardless of the norm type used.

Table 5.1 Lagrange multipliers of the post-contingency power balance relations as a function of the load

		$\boldsymbol{\mu}(k)$ (\$/MWh)					
Load range (MW)	Bus	1	2	3	4	5	6
	1	0	0	5	0	0	0
$[0, 50]$	2	0	0	5	0	0	0
	3	0	0	5	0	0	0
	1	2	0	5	0	0	0
$(50, 100]$	2	2	0	5	0	0	0
	3	2	0	5	0	0	0
	1	7	5	0	0	0	0
$(100, 105]$	2	7	5	0	0	0	0
	3	7	5	0	0	0	0
	1	7	0	0	-3	0	-2
$(105, 116.6)$	2	7	0	0	0	0	-2
	3	7	0	0	0	0	13

By inspection, in the range of demand from 0 to 50 megawatts, the vector $\boldsymbol{\mu}(k)$ is nonzero for $k = 3$ only. This means that the set of umbrella contingencies is composed of the contingency corresponding to the failure of generator 3 only. In this range, all the energy is generated by generator 3, which is the cheapest incrementally as seen in Table E.2. Therefore, to meet the loss of that generator, an equal amount of up-spinning reserve is provided by generator 1, the cheapest provider of up-spinning reserve. The Lagrange multipliers at the three buses are all equal to \$5 per megawatt-hour, which is the offered marginal cost of up-spinning reserve of generator 1.

Next, in the demand range between 50 and 100 megawatts, $\boldsymbol{\mu}(k)$ is nonzero for $k = 1$ and $k = 3$. These are the contingencies corresponding to the failures of generators 1 and 3 respectively. Here, generator 3 produces at its maximum of 50 megawatts, while generator 1 supplies the remaining demand. To cover the loss of generator 3, generators 1 and 2 provide together at least 50 megawatts of up-spinning reserve with generator 2 supplying enough up-spinning reserve to cover the loss of generator 1. For instance, when the load is 70 megawatts, generator 1 produces 20 megawatts while it provides 50 megawatts of up-spinning reserve to cover for the loss of generator 3. Concurrently, generator 2 provides the 20 megawatts of reserve required to cover for the loss of generator 1.

In the following demand range running between 100 and 105 megawatts, the failure of generator 3 leaves the set of umbrella contingencies while that of generator 2 enters. Here, generator 3 still produces at its maximum capacity, while generator 1 picks up the residual load. Now, generator 2 provides all the up-spinning reserve required to replace either generators 1 or 3, but it generates no energy. Because the pre-contingency generation level of generator 1 exceeds that of generator 3, the reserve provision of generator 2 is therefore set by generator 1. This explains why generator 3 has exited the umbrella set. In addition, the nonzero Lagrange multiplier vector associated with the failure of generator 2 indicates that if generator 2 were to have to generate some nonzero level of energy (if, for instance, $g_2^{\min} > 0$), then generator 1 would need to provide that corresponding level of power as up-spinning reserve at the rate of \$5 per megawatt-hour.

In the last range of load for which there exists a feasible schedule (for a demand in excess of 116.6 megawatts there is no feasible solution to this SCOPF problem), generator 1 keeps being a member of the umbrella set while the failures of line 1 ($k = 4$) and line 3 ($k = 6$) enter the umbrella set. Generators 1 and 3 are still the only producers of energy in the pre-contingency state, while generator 2 provides 55 megawatts of up-spinning reserve to counter the failures of generators 1 and 3, independently of the load. In the current case, demand-side up-spinning reserve has to be scheduled to cover the loss of generator 1 and the losses of lines 1 and 3. Furthermore, generator 1 needs to supply some down-spinning reserve so as to make it possible to meet the line flow limitations following any of the credible line failures. We note, moreover, that the umbrella set here is not unique as the failure of line 2 ($k = 5$) produces an identical effect as that of the loss of line 3. As a result, we see that as long as one of these two contingencies is included in the umbrella set, the social cost of the relaxed SCOPF will be identical to that of the initial full SCOPF. We point out also that in the case of the line failures here, the associated Lagrange multipliers are not all equal over all buses; this indicates the presence of line congestion in the corresponding post-contingency states. The negative signs of some of the multipliers associated with the failures of the lines reflect that to relieve the line congestion in the post-contingency states, it would be economically favorable if there were some loads located at buses 1 and 2.

We can also see from Table 5.1 that there is no "super-umbrella" contingency, whose membership in the set of umbrella contingencies is unaffected by load variations.

Finally, we see that there are generally only a few umbrella contingencies. In the current example, when the grid can be reduced to a single node for the pre- and all the credible post-contingency states (that is, within the load range $[0, 105]$ megawatts when there is no pre- or post-contingency congestion), there are at most two umbrella contingencies that correspond to the losses of one of the two generators producing the largest aggregated amounts of power and up-spinning reserve. This observation is not exceptional to the current example wherein two generators are providing the up-spinning reserve dedicated to cover the failure of one another. This observation is the general theoretical basis for setting the up-spinning reserve requirement equal to the capacity of the largest generator in network-free deterministic reserve-constrained market-clearing formulations.

5.4.2 Stochastic security-constrained market-clearing

The second simulation example studies how the membership of the set of umbrella contingencies is affected by varying the norm type for ranking and cutting-off contingencies in stochastic-based SCOPF problems. The formulation of the SCOPF is the same as that of Section 5.4.1 except for: (i) the objective function now measures the expected social cost in both the pre- and post-contingency states

$$\min p(0)[\mathbf{a}^T\mathbf{g} + (\mathbf{q}^{up})^T\mathbf{r}^{up} + (\mathbf{q}^{dn})^T\mathbf{r}^{dn} + (\mathbf{q}_d^{up})^T\mathbf{r}_d^{up} + (\mathbf{q}_d^{dn})^T\mathbf{r}_d^{dn}] \\ + \sum_{k\in\mathcal{K}} p(k)[\mathbf{a}^T\mathbf{g}(k) + \mathbf{v}^T\mathbf{l}(k)], \quad (5.10)$$

where $\mathbf{v}$ is the vector of the nodal value of lost load—equal to \$500 per megawatt-hour for all buses; (ii) the post-contingency bus power balance conditions now take into account possible load shedding $\mathbf{l}(k)$

$$\mathbf{H}(\mathbf{g}(k), \mathbf{d}(k), \boldsymbol{\delta}(k), \mathbf{l}(k), k) = \mathbf{0}; \quad \forall k \in \mathcal{K}, \quad (5.11)$$

where the vector variables $\mathbf{g}(k)$ and $\mathbf{d}(k)$ correspond to the re-dispatch of generation and demand through reserve deployment, respectively, and $\boldsymbol{\delta}(k)$ are the post-contingency bus voltage angles; and, (iii) associated limits on the load shedding variables

$$\mathbf{0} \leq \mathbf{l}(k) \leq \mathbf{d}(k); \quad \forall k \in \mathcal{K}. \tag{5.12}$$

In addition, the contingency state probabilities $p(0)$ and $p(k)$ are calculated (according to the principles found in Appendix A.1) from the forced-outage rates given in Appendix E.1, assuming that the six random contingencies occur independently. The calculated probabilities are given in Table 5.2. We notice that the probabilities reported in Table 5.2 do not sum up to one since we did not consider the failure modes involving multiple-equipment contingencies.

Table 5.2 Contingency probabilities

k	0	1	2	3	4	5	6
$p(k)$ (10^{-4})	8645	455	176	651	4	4	4

For a demand of 110 megawatts at bus 3, the corresponding optimal schedule is reported in Table 5.3. Also, the expected values of the involuntary load shedding actions associated with each of the contingencies are found in Table 5.4. From Table 5.3, we see that the 60 megawatts of up-spinning reserve supplied by generator 2 can cover the failures of both generators 1 and 3, and the 5 megawatts of down-spinning reserve provided by generator 1 is required to meet the steady-state line flow limits if any one of the transmission lines fails.

Table 5.3 Pre-contingency generation, demand and reserve schedule for the full SCOPF

Generator i	g_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)
1	60	0	5
2	0	60	0
3	50	0	0
Demand	d (MW)	r_d^{up} (MW)	r_d^{dn} (MW)
	110	0	0

The optimal value of the expected social cost is \$3265.73. We notice that the much cheaper involuntary load shedding actions are used in place of demand-side reserve when

Table 5.4 Expected involuntary load shed at bus 3 following contingencies

k	1	2	3	4	5	6
$p(k)l(k)$ (kWh)	0	0	0	0	2	2

either line 2 or 3 is lost. Indeed, we note here that the corresponding expected cost of involuntary load shedding is quite small (4×10^{-3}MWh$\times$\$500/MWh = \$2.00) in comparison to that of scheduling and deploying demand-side reserve (0.8645×5 MWh $\times$ \$20/MWh = \$86.45).

Next, the first three rows of Table 5.5 report the Lagrange multiplier vectors corresponding to the power balance relations under the six credible contingencies. The last three rows show respectively the one, two and infinity norms of these vectors.

Table 5.5 Lagrange multipliers of the post-contingency power balance relations and their corresponding norms

	$\mu(k)$ (\$/MWh)					
Bus	1	2	3	4	5	6
1	7.87	4.85	1.95	−4.31	0.01	0.01
2	7.87	4.85	2.60	0.02	0.02	0.01
3	7.87	4.85	3.25	0.02	0.22	0.22
$\|\cdot\|_1$	23.61	14.55	7.80	4.35	0.25	0.24
$\|\cdot\|_2$	13.63	8.40	4.60	4.31	0.22	0.22
$\|\cdot\|_\infty$	7.87	4.85	3.25	4.31	0.22	0.22

Table 5.5 demonstrates that the various vector norms lead to different rankings of the contingencies. For the one and two norms, the ordering is $\{1, 2, 3, 4, 5, 6\}$ whereas in the case of the infinity norm it is $\{1, 2, 4, 3, 5, 6\}$. For all three norms considered, however, the line outage contingencies $k = 5$ and 6 (corresponding to the failures of lines 2 and 3 respectively) are seen to have very little impact on the solution of the SCOPF. In fact, omitting these two contingencies and solving the corresponding relaxation of the SCOPF, we find that the optimal pre-contingency schedule is identical to that in Table 5.3. As anticipated, the expected social cost has diminished slightly to \$3264.54—a difference of \$1.19 or 0.04% from the expected social cost associated with the full SCOPF. We explain this from the absence of the expected costs of reserve deployment and involuntary load shedding actions

corresponding to these contingencies in the objective function of the relaxed problem. Thus, there is a clear indication here that these two contingencies are not umbrella contingencies.

Now, we investigate the scheduling results obtained if the stochastic SCOPF is relaxed even more. For instance, if the cutoff rule of Proposition 5.1 uses the one norm and some $\varepsilon >$ \$4.35 per megawatt-hour, the umbrella set would be $\mathcal{U} = \{1, 2, 3\}$. On the other hand, if we were to use the ranking based on the infinity norm with some $\varepsilon >$ \$3.25 per megawatt-hour, the umbrella set would then be different with $\mathcal{U} = \{1, 2, 4\}$.

If we let the set of umbrella contingencies be $\mathcal{U} = \{1, 2, 4\}$, the pre-contingency schedule obtained for the relaxed SCOPF is the same as the one found with the full set of contingencies in Table 5.3. Here, however, we remark that for this relaxed SCOPF no involuntary load shedding actions are required. We recall from Table 5.4 that involuntary load shedding was expected for the full SCOPF solution for contingencies $k = 5$ (loss of line 2) and 6 (loss of line 3), which were omitted here. As a result of the omissions of contingencies $k = 3, 5$ and 6, the expected cost has gone down to \$3240.60—a difference of \$25.13 or 0.77% from the expected social cost corresponding to the full SCOPF.

In the alternative where we use the umbrella set $\mathcal{U} = \{1, 2, 3\}$, we get the different pre-contingency schedule shown in Table 5.6. As with the case with the umbrella set $\mathcal{U} = \{1, 2, 4\}$, we find that there is no scheduled involuntary load shedding here. Notwithstanding the fact that the pre-contingency schedule obtained with the smaller set of contingencies differs from that of the full SCOPF (unlike with $\mathcal{U} = \{1, 2, 4\}$), the change in the expected social cost is less—the expected cost now equals \$3263.90, a decrease of \$1.83 or 0.06% from the expected social cost of the full SCOPF.

Table 5.6 Pre-contingency generation, demand and reserve schedule with the umbrella set $\mathcal{U} = \{1, 2, 3\}$

Generator i	g_i (MW)	r_i^{up} (MW)	r_i^{dn} (MW)
1	57.5	0	0
2	2.5	55	0
3	50.0	0	0
Demand	d (MW)	r_d^{up} (MW)	r_d^{dn} (MW)
	110	2.5	0

It could be argued that the "quality" of the umbrella set $\mathcal{U} = \{1, 2, 4\}$ is superior to that of $\mathcal{U} = \{1, 2, 3\}$ because it has the ability to cover a wider range of credible contingencies. We mean that, by including contingency $k = 4$, the SCOPF solution schedules the necessary down-spinning reserve at bus 1 required to limit the post-contingency congestion after any one of the line failures. Still, given the relative values of the forced-outage rates of the equipments, we remark that generator failures are clearly more likely than line failures. One can therefore argue in favor of the alternate umbrella set, $\mathcal{U} = \{1, 2, 3\}$, because its members have higher probabilities of occurrence, and also have a lesser impact on the optimal value of the objective function, as required by Definition 5.1.

Lastly, given that the contingency ranking is based on the infinity norm, the contingency $k = 4$ is said to have a high impact on the solution of the SCOPF even though its impact is substantial at bus 1 only. Yet, contingency $k = 3$ is found to be more important by the one and two norms. In this case, we clearly see the network-wide "averaging" function performed by the one and two vector norms already alluded to in Section 5.3. This ranking behavior requires from the user of the identification scheme to be aware of which type of contingency the umbrella set should consist of: either contingencies imposing network-wide effects or contingencies for which the localized effects are the most important.

5.5 Summary

In this chapter, we defined with rigor the notion of the set of umbrella contingencies for security-constrained optimal power flow problems in both deterministic and probabilistic forms. We proposed an original method to identify the members of the set of umbrella contingencies from the Lagrange multipliers associated with the post-contingency power balance relations of the SCOPF problems. As part of this method, we suggested a heuristic rule to rank the credible contingencies according to their marginal economic impact on the optimal solution of the SCOPF, based on the vector norms of the Lagrange multiplier vectors. Following the ranking step, it is then possible to cut off those lowly-ranked contingencies.

A first numerical study looked at how the range of validity of the set of umbrella contingencies changes as the system demand is varied in the case of a deterministic SCOPF. The main observations were that, generally, there are only a few umbrella contingencies, especially in cases when the grid can be reduced to a single node—in which case there are at most two umbrella contingencies. Moreover, this study revealed that the set of umbrella contingencies may not be unique when some of the contingencies are umbrella to one another.

The second case study examined the impacts of using ranking and cutoff schemes to heuristically determine the umbrella set for stochastic SCOPF problems. In this study, we demonstrated that the pre-contingency schedules of the SCOPF solved subject to a reduced set of umbrella contingencies can be identical or very close to the pre-contingency schedule of the full SCOPF. In addition, this example illustrated how the type of the vector norm used in ranking the contingencies may favor contingencies with network-wide effects or those with important but more localized impacts. Such flexibility may be welcome from the users' point of view because the method leaves him or her with some latitude in controlling the membership in the set of umbrella contingencies, while the proposed method still provides for increased rigor in the classification of the contingencies.

In the larger context of the present dissertation, the notion of umbrella contingencies is clearly of interest since it provides yet another potential means to reduce the dimensions of security-constrained electricity market-clearing problems. As presented in this chapter, the proposed ranking and cutoff scheme is intended to provide electricity market operators with an *a priori* estimation of the set of umbrella contingencies so to simplify the resolution of the market-clearing problems. As this estimation should generally be based on predicted network and economic information, the proposed method is bound to neglect some of the true umbrella contingencies and to retain contingencies that, at the moment when uncertainty is revealed, actually lie under the umbrella of other more severe contingencies. Nevertheless, we believe that the specification of a *reasonable* set of umbrella contingencies should constitute a good compromise between having to consider too many or no contingencies at all.

Chapter 6

Conclusions

Aquí podemos, hermano Sancho Panza,
meter las manos hasta los codos en esto
que llaman aventuras.

Don Quijote de la Mancha I
Miguel de Cervantes, 1547–1616

6.1 Dissertation Overview

We proposed and formulated an electricity market-clearing system integrating a stochastic security criterion. We based this security criterion on a probabilistic measure of the expected load not served consequent to the occurrence of pre-selected sets of generator and line random failures and load disturbances. We demonstrated that the required levels of the different reserve services can be implicitly determined by economically penalizing the operation of the market through the associated demand-side costs of involuntary load shedding. Under this approach, the market-clearing problem has the flexibility to balance the respective expected costs of: pre-contingency preventive security control actions including unit commitment, dispatch and reserve scheduling decisions; post-contingency corrective actions based on reserve deployment measures that include further unit commitment decisions and load-generation re-dispatch; and, post-contingency involuntary load shedding. We furthermore addressed several practical implementation issues particular to the proposal, which include: (i) multi-period unit commitment; (ii) the treatment of transmission congestion; (iii) the proposal's categorization within the larger class of stochastic optimization problems; (iv) the formulation's computational complexity and possible relief measures; (v) the uncertainties in the equipment statistical failure data and consumers' value of lost load; and, (vi) the integration of the proposal within the larger power system operations paradigm. We remarked that this proposal contrasts the current, essentially deterministic, industry operations planning philosophy and practices. Case studies showed, however, that by going against this traditional conservative philosophy, electricity market-clearing with stochastic security can lead to non-negligible economic savings for society, while ensuring that consumers still enjoy a secure supply of electricity in light of their valuation of load shedding.

Next, we demonstrated a number of theoretical results pertaining to the prices of energy and security derived from the optimal solutions of market-clearing with stochastic security. We showed, among other things, that involuntary load shedding should be used in the aftermath of a contingency if and only if the expected marginal costs of scheduling reserves and deploying them after that contingency are greater than the expected marginal costs of load shedding. We also established that, in a linear market-clearing formulation, when load is shed it is the last of the post-contingency recourses used by the system operator to balance power at any given bus.

We went on to extend the model of electricity market-clearing with stochastic security to propose a day-ahead electricity market-clearing formulation capable of accounting for non-dispatchable intermittent power generation sources. Because of its prominent importance in current and future electricity grids, we assumed that wind power generators constituted the entire non-dispatchable generation capacity installed in the power system. The resulting electricity market-clearing model was shown to take into account uncertainties in the next day wind power generation and demand predictions. Also, we demonstrated how the market-clearing formulation can integrate the scheduling of large-scale centralized energy storage equipments. Being able to handle such storage facilities is key to the commercial success of wind and other intermittent, mostly renewable, generation resources. As proposed in Chapter 2, the market-clearing formulation here was shown to balance the expected costs associated with reserve scheduling and deployment against those of involuntary load shedding actions given the probabilities of the different load-wind scenarios considered.

Lastly, we rigourously defined the concept of the set of umbrella contingencies for security-constrained optimal power flow problems, a class of power system scheduling problems to which electricity market-clearing with stochastic security belongs. We derived an identification method to discover the members of the set of umbrella contingencies by making use of the vector norms of the Lagrange multipliers associated with post-contingency power balance relations. We showed how this identification method is applicable to both deterministic and stochastic security-constrained market-clearing problems. As an extension of this identification method, we suggested a heuristic contingency ranking and cutoff rule based on the contingencies' Lagrange multiplier vector norms. We showed how these umbrella contingency identification and ranking methods could be of use to system operators when specifying reduced sets of contingencies required to simplify security-constrained market-clearing problems.

6.2 Recommendations for Future Work

Several recommendations for further investigation were already made throughout the dissertation. We summarize them here along with other promising research directions.

1. The existing formal scenario reduction methods [121–123] for stochastic programming problems should be investigated for the stochastic market-clearing problems defined in this dissertation. These techniques are without a doubt necessary in building and extracting those scenarios which can represent the widest spectrum of probable states of nature, while keeping their number within reasonable levels so as to limit computational efforts. In relation with this aspect, one can probably conjecture interesting parallel concepts existing between those of umbrella contingencies and reducible scenarios. We note here that this recommendation applies to market-clearing models where equipment failures are the primary sources of uncertainty as much as to those where it is the uncertainty in the generation output that is the dominant factor.
2. Decomposition methods should be investigated as possible solution strategies for electricity market-clearing with stochastic security. Benders' decomposition, which is often associated with the solution of large-scale stochastic optimization problems [85, 116], as well as "branch-and-price" decomposition, which is, on the other hand, mostly associated with the solution of large mixed-integer optimization problems [118–120], are surely the most likely candidates applicable to the problems found in

this dissertation. One advantage the decomposition methods have over the classic branch-and-cut algorithms for mixed-integer linear programming is their ability to break apart large-dimension problems and resort to iterative solutions of smaller sub-problems. This ability to breakdown problems into smaller pieces can be critical in large-scale instances especially if there are computer memory limitations. Also, these methods have the added advantage that they can be implemented on parallel computers that simultaneously work out solutions to the decomposed sub-problems. This can lead us to conjecture that the exploitation of the computational parallelism could generate some CPU time savings. In fact, the essential aspect that remains to be determined is which of the two decomposition methods, Benders' or "branch-and-price"—or even some combination of the two—, is the most appropriate to solve quickly and efficiently realistically-sized security-constrained market-clearing problems.

3. Investigating the inclusion of risk measures in a formulation of the electricity market-clearing with stochastic security is of practical interest. For instance, it would be reasonable for market operators to minimize expected social costs while, at the same time, making sure that the variance of that cost or the variance of the associated cost of load shedding remains accordingly small. The classical way to achieve this goal is through the addition of a term in the objective function of the market-clearing proportional to the conditional value-at-risk of the quantity whose variance has to be kept small [193].
4. Investigative efforts are needed to develop generation-side offering and demand-side bidding strategies in the context of an electricity market whose schedules are found through market-clearing with stochastic security. The main challenge here stems from the fact that generators and demands, unlike the system operator, would generally have incomplete information about the probabilities of the various uncertainty scenarios.
5. Offering and bidding strategies are obviously dependent on the financial settlement methods used to remunerate the different products and services traded in the market. In this dissertation, we did not specify nor did we study the impacts such rules. The focus of this future line of investigation should be on the various welfare properties induced by the settlement schemes. These properties include the expected generator profits and consumer payments as well as their variances. These settlement rules

could be based solely on expected prices (as calculated in Chapters 2 and 3), or on the real-time prices that correspond to the realization of given scenarios. Likewise, another problem to be studied is whether settlements should be made using day-ahead prices only, real-time prices only or a combination of the two.

6. When scheduling a power system considering uncertain wind power generation, the normality assumption of the probability distribution of the prediction error may not always be valid. This would be the case in situations when the wind power generators in a given system are not evenly-distributed geographically or when there are only a few large wind power generators. An investigation of effects of non-normal prediction error distributions on market-clearing results is thus warranted.
7. The wind power generation scenarios constructed in this dissertation assumed that the inter-period wind generation levels were fully uncorrelated. This assumption, however, is generally not true in practice. Research efforts should be devoted to study how the level of inter-period correlation can improve the economic efficiency and the reliability of the power system. The corresponding computational effort should be reassessed too because the consideration of inter-period correlations may no longer require the consideration of those scenarios containing very unlikely inter-period wind power generation transitions.
8. The impacts on market-clearing results of prediction errors of other intermittent generation resources (like photovoltaic power, for instance) should be studied. Of course, as long as installed capacities of these alternative resources remain marginal, their corresponding impacts should also remain marginal. Nonetheless, building scenarios that model their joint random behavior with that of wind power represents probably the most important research challenge here.
9. The investigation of end-of-period policies for energy storage systems is yet another interesting research opening. It remains unclear whether the best strategy is to assign a value—and in fact, the question is also "What value?"—to the energy stored or to require simply that the expected value of the energy stored be within some, probably conservative, pre-defined range.
10. It is to be seen whether the stochastic market-clearing formulation with wind power generation uncertainty, originally destined to conduct short-term generation and reserve scheduling, is expandable to conduct stochastic generation planning studies over medium- to long-term time horizons. Surely here, some simplifications have to

be introduced; for instance, hydrothermal unit commitment and ramping limitations do not need to be modelled explicitly over month-long planning studies. Likewise, some other longer-term factors like seasonality would have to be introduced.

11. Combining the notion of umbrella contingencies with statistical methods may generate interesting research directions. Given a large number of records of the prevailing operating conditions when some contingency was part of the set of umbrella contingencies, one could derive a number of statistical indices related to that contingency. For instance, one such index could be the probability that a contingency is in the umbrella set conditional to the realization of a specific set of operating conditions.

Appendix A

Equipment Failure Probabilities

This appendix provides the basic derivations of the failure probabilities of equipments used in this dissertation. Below, we divide the analysis into time-static and time-dynamic probability calculations.

A.1 Time-Static Probabilities

For time-static problems in this dissertation, we model the occurrence of the random failure of some component (generator, transmission line, transformer, etc.) k as a Bernoulli random variable. Thus, if component k fails the random variable ξ_k equals 0; otherwise, it equals 1. The uncertainty in this state of nature is defined by the associated forced-outage rate [46] of that component, U_k.[1] We assume here that the forced-outage rates of components in the network are known from historical failure data collected by an independent monitoring agency. In Canada, for example, the Canadian Electricity Association has been compiling information about individual line and generator failures since 1977 [55, 56].

Therefore, we have for each network component $k = 1, \ldots, K$

$$U_k = P[\xi_k = 0]. \tag{A.1}$$

[1]The expression "forced-outage rate" is somewhat a misnomer since it is not a rate. A more appropriate term could be "relative unavailability" as this dimensionless quantity measures the per unit time a given piece of equipment is out of service due to random failures.

Also, we can derive the associated "relative availability," A_k, of components $k = 1, \ldots, K$

$$A_k = 1 - U_k = P[\xi_k = 1]. \tag{A.2}$$

Next, assuming that random failures happen independently, we can derive the probabilities of random events involving the entire set of network components [46, 168]. For example, we have:

All components are available

$$p(0) = \prod_{k=1}^{K} A_k. \tag{A.3}$$

Component k is out

$$p(k) = U_k \prod_{\substack{z=1 \\ z \neq k}}^{K} A_z, \tag{A.4}$$

and so on.

A.2 Time-Dynamic Probabilities

In classical reliability theory, the time of failure τ of a given piece of equipment is modeled as an exponentially-distributed random variable [46, 168]. In this section, we show basic probability calculations for contingencies which may include the simultaneous loss of several components, but exclude contingencies made up of sequential failures. For example, the random event "*contingency k occurs within the time interval τ*," denoted by $X(k, \tau)$, has the probability [168]

$$P[\,X(k,\tau)\,] = \int_{\tau-1}^{\tau} \lambda_k e^{-\lambda_k \zeta}\, d\zeta = e^{-\lambda_k \tau}(e^{\lambda_k} - 1). \tag{A.5}$$

The parameter λ_k represents the reciprocal of the mean time to the occurrence of contingency k, a quantity estimated from historical data [55, 56].

We moreover remark that since repair times are usually longer than the 24-hour scheduling horizon of most day-ahead electricity markets considered in this dissertation, repairs are ignored so that once some equipment fails it is assumed to be unavailable for the remainder of the horizon.

In addition, the probability of the random event "*contingency k does not occur during the scheduling horizon,*" denoted by $Y(k)$, is

$$P[Y(k)] = 1 - \int_0^T \lambda_k e^{-\lambda_k \zeta}\, d\zeta = e^{-\lambda_k T}. \tag{A.6}$$

The probability $p(0)$ that none of the pre-selected contingencies occur during the scheduling horizon can now be calculated from the atomic random events $Y(k)$

$$p(0) = \prod_{k=1}^{K} P[Y(k)] = \prod_{k=1}^{K} e^{-\lambda_k T}. \tag{A.7}$$

Likewise, $p(k,\tau)$, the probability that contingency k occurs during the interval τ given that all other system components are available for the entire scheduling horizon, is

$$p(k,\tau) = P[X(k,\tau)] \prod_{\substack{z=1 \\ z\neq k}}^{K} P[Y(z)] = e^{-\lambda_k \tau}(e^{\lambda_k} - 1) \prod_{\substack{z=1 \\ z\neq k}}^{K} e^{-\lambda_z T}. \tag{A.8}$$

We note that in deriving the above probabilities, the assumption is that the random occurrence of the pre-selected contingencies are statistically independent, and that since this set is not exhaustive, the probabilities $p(0)$ and $p(k,\tau)$ do not sum up to one.

Appendix B

Feasible Operational Regions of Generators and Demands

This appendix is devoted to the thorough description of constraints bounding the feasible operating regions of generators and demands.

In the main body of the dissertation, the feasible operating region of a generator i during period t is denoted compactly by the symbol $\mathcal{G}_{it}$. This set restricts, for both the pre- and post-contingency states:

- The on/off status of the generator, which is governed by its minimum up- and down-time limits;
- The power output of the generator, which is restricted by its minimum and maximum generation limits as well as inter-temporal ramping constraints; and,
- The amounts of the different reserve services which can be provided by the generator.

This appendix covers the first two items only. The third one is covered fully in Appendix C.

Likewise, the feasible operating region of demand at bus m during period t is denoted compactly in the main body of the dissertation by the symbol $\mathcal{D}_{mt}$. The set $\mathcal{D}_{mt}$ limits, for both the pre- and post-contingency states:

- The power consumption of the demand, which is limited by its elasticity limits; and,
- The amounts of the different reserve services which can be provided by the demand.

Here we cover the first item only. The second one is covered fully in Appendix C.

Throughout this work, the basic generation constraints, those enforcing minimum up-and down-time as well as those constraining capacity and unit ramping, were those developed by Ruiz-Peinado in [79]. This formulation has proven to be far superior to other classical unit commitment formulations [45,68–78,80] since it requires a much lower number of binary variables.

In fact, the Ruiz-Peinado formulation only uses the regular on/off variables u_{it}, whereas other formulations use, on top of the usual on/off variables, binary variables to model the startup (generally denoted by the variable y_{it}, where $y_{it} = 1$ if unit i starts up at the beginning of period t or equals 0 otherwise) and shutdown (generally denoted by the variable z_{it}, where $z_{it} = 1$ if unit i shuts down at the beginning of period t or equals 0 otherwise) of the generating units. Thus, for a system containing I generating units and for a scheduling horizon of T periods, the Ruiz-Peinado formulation uses one third of the binary variables, without the addition of any extra continuous variables or constraints [79]. This formulation represents a significant computational improvement, reducing significantly the core memory requirements of a given problem as well as reducing the upper bound on the size of the branch-and-cut tree that may be generated by the mixed-integer linear programming solver (see Appendix F for more details).

Sections B.1 and B.2 below outline respectively the details of the minimum up- and down-time constraints and those of the generator capacity and ramping limitations. Finally, Section B.3 gives the details of demand-side bidding elasticity limitations.

B.1 Generation Minimum Up- and Down-Time Constraints

The following parameters are required to define the generator minimum up- and down-time limitations:

u_{i0}	Commitment state of generator i during period 0.
UT_{i0}	Cumulative up-time of generator i at the beginning of period 0.
DT_{i0}	Cumulative down-time of generator i at the beginning of period 0.
MUT_i	Minimum up-time of generator i.
MDT_i	Minimum down-time of generator i.

B.1.1 Minimum up-time

For all generators $i = 1, \ldots, I$, we define the quantity $P_i = \min\{T,\ (MUT_i - UT_{i0})u_{i0}\}$ specifying the number of hours that a unit must stay on from the beginning of the scheduling horizon. Thus, for all $i = 1, \ldots, I$ the minimum up-time constraints are given from [79]

$$\sum_{j=1}^{P_i}(1 - u_{ij}) = 0, \tag{B.1}$$

$$\sum_{j=1}^{MUT_i} u_{ij} - MUT_i(u_{it} - u_{i0}) \geq 0; \qquad t = 1,\ P_i = 0, \tag{B.2}$$

$$\sum_{j=t}^{t+MUT_i-1} u_{ij} - MUT_i(u_{it} - u_{i(t-1)}) \geq 0; \quad t \neq 1,\ P_i + 1 \leq t \leq T - MUT_i + 1, \tag{B.3}$$

$$\sum_{j=1}^{T}(u_{ij} - u_{it} + u_{i0}) \geq 0; \qquad t = 1,\ T - MUT_i + 2 = 1, \tag{B.4}$$

$$\sum_{j=t}^{T}(u_{ij} - u_{it} + u_{i(t-1)}) \geq 0; \qquad t \neq 1,\ T - MUT_i + 2 \leq t \leq T. \tag{B.5}$$

B.1.2 Minimum down-time

For all generators $i = 1, \ldots, I$, we define the quantity $Q_i = \min\{T,\ (MDT_i - DT_{i0})(1 - u_{i0})\}$ specifying the number of hours that a unit must stay off from the beginning of the scheduling horizon. Thus, for all $i = 1, \ldots, I$ the minimum down-time constraints are given from [79]

$$\sum_{j=1}^{Q_i} u_{ij} = 0, \tag{B.6}$$

$$\sum_{j=1}^{MDT_i} (1 - u_{ij}) - MDT_i(u_{i0} - u_{it}) \geq 0; \qquad t = 1,\ Q_i = 0, \tag{B.7}$$

$$\sum_{j=t}^{t+MDT_i-1} (1 - u_{ij}) - MDT_i(u_{i(t-1)} - u_{it}) \geq 0; \quad t \neq 1,\ Q_i + 1 \leq t \leq T - MDT_i + 1, \tag{B.8}$$

$$\sum_{j=1}^{T}(1 - u_{ij} - u_{i0} + u_{it}) \geq 0; \qquad t = 1,\ T - MDT_i + 2 = 1, \tag{B.9}$$

$$\sum_{j=t}^{T}(1 - u_{ij} - u_{i(t-1)} + u_{it}) \geq 0; \qquad t \neq 1,\ T - MDT_i + 2 \leq t \leq T. \tag{B.10}$$

We note that the above set of constraints, (B.1)–(B.10), applies equally to the post-contingency unit commitment variables, $u_{it}(k,\tau)$.

B.2 Generation Ramping and Output Capacity Constraints

The following parameters are required to define the generator capacity and ramping limitations:

u_{i0}	Commitment state of generator i during period 0.
g_{i0}	Power output of generator i during period 0.
$g_i^{\min}$	Minimum power output of generator i.
$g_i^{\max}$	Maximum power output of generator i.
R_i^{dn}	Ramp-down limit of generator i.
R_i^{up}	Ramp-up limit of generator i.
R_i^{sd}	Shutdown ramp limit of generator i.
R_i^{su}	Startup ramp limit of generator i.

The bounds on the available power are, for all the generators $i = 1, \ldots, I$, [79]

$$g_i^{\min} u_{it} \leq g_{it} \leq g_i^{\max} u_{it}; \quad t = 1, \ldots, T. \tag{B.11}$$

The up-ramping limitations are, for generators $i = 1, \ldots, I$,

$$g_{it} \leq g_{i(t-1)} + R_i^{up} u_{i(t-1)} + R_i^{su}(u_{it} - u_{i(t-1)}) + g_i^{\max}(1 - u_{it}); \qquad t = 2, \ldots, T, \tag{B.12}$$

$$g_{it} \leq g_{i0} + R_i^{up} u_{i0} + R_i^{su}(u_{it} - u_{i0}) + g_i^{\max}(1 - u_{it}); \qquad t = 1. \tag{B.13}$$

Furthermore, down-ramping constraints are imposed on every generator $i = 1, \ldots, I$

$$g_{it} \geq g_{i(t-1)} - R_i^{dn} u_{it} - R_i^{sd}(u_{i(t-1)} - u_{it}) - g_i^{\max}(1 - u_{i(t-1)}); \qquad t = 2, \ldots, T, \tag{B.14}$$

$$g_{it} \geq g_{i0} - R_i^{dn} u_{it} - R_i^{sd}(u_{i0} - u_{it}) - g_i^{\max}(1 - u_{i0}); \qquad t = 1. \tag{B.15}$$

Note as well that the above set of constraints, (B.11)–(B.15), applies equally to the post-contingency generator power output variables, $g_{it}(k,\tau)$.

B.3 Demand Elasticity Limits

As they bid in the electricity market, the demands have the opportunity to specify lower and upper limits on their consumption. These bounds are often denoted as the "elasticity limits" [10]. In this dissertation, the demand elasticity limits apply only to their pre-contingency power consumption. The post-contingency demand limits, on the other hand, are governed by the up- and down-going spinning reserve bounds described in Appendix C.

Mathematically, the demand elasticity limits are expressed as, for $m = 1, \ldots, M$ and $t = 1, \ldots, T$

$$d_{mt}^{\min} \leq d_{mt} \leq d_{mt}^{\max}, \tag{B.16}$$

where $0 \leq d_{mt}^{\min} \leq d_{mt}^{\max}$ are parameters submitted as part of the demand-side bids. Note that in the case of inelastic demand, the two limits are set to be equal to each other, $d_{mt}^{\min} = d_{mt}^{\max} = d_{mt}$.

Appendix C

Reserve Determination Constraints

This appendix describes the constraints affecting the reserve services (spinning and non-spinning as well as up- and down-going) offered by generators and demands.

C.1 Spinning Reserve

C.1.1 Generation-side

The provision of generation-side up-spinning reserve is restricted as

$$0 \leq r_{it}^{up} \leq r_{it}^{up\ \max} u_{it}, \tag{C.1}$$

for all generators, $i = 1, \ldots, I$, and for all time periods, $t = 1, \ldots, T$. The parameters $r_{it}^{up\ \max}$ are the upper limits on the up-spinning reserve offers imposed by each of the generators. Clearly, up-spinning reserve can be provided by generator i during period t only if it is on, that is, if $u_{it} = 1$.

Here, the up-spinning reserve provided by generator i during period t is the largest among all the pre-selected contingencies, k, and failure times, τ, of the difference between its post- and pre-contingency generation levels. Mathematically, this condition translates into the linear inequalities for $k = 1, \ldots, K$ and $\tau = 1, \ldots, T$

$$r_{it}^{up} \geq g_{it}(k, \tau) - g_{it} - g_i^{\max}(2 - u_{it} - u_{it}(k, \tau)), \tag{C.2}$$

where $g_i^{\max}$ is the maximum power output of generator i.

To see how (C.1) and (C.2) set the reserve levels in a consistent manner, we examine the possible cases which may arise.

First, we assume that $u_{it} = 0$. From (C.1), r_{it}^{up} should equal zero. To demonstrate that (C.2) is consistent with that condition, consider the case where the post-contingency commitment variable, $u_{it}(k, \tau)$ is equal to 1—meaning that generator i has been turned on in response to a contingency. Then, (C.2) requires that $r_{it}^{up} \geq g_{it}(k, \tau) - g_i^{\max}$, whose right-hand side is less than or equal to zero, and is consistent with the lower bound in (C.1). A similar argument applies if $u_{it}(k, \tau) = 0$.

When $u_{it} = 1$, the up-spinning reserve provided by generator i during period t lies in the range $[0, r_{it}^{up\,\max}]$ as required by (C.1). The actual level of reserve is set from (C.2), considering all the contingencies. The case when $u_{it}(k, \tau) = 0$ does not impose any constraint on the reserve; however, if $u_{it}(k, \tau) = 1$, inequality (C.2) requires that $r_{it}^{up} \geq g_{it}(k, \tau) - g_{it}$ for all k and τ. In other words, the up-spinning reserve provided by generator i has to be larger than or equal to the largest of the post-contingency up-going power production deviation away from the pre-contingency dispatch, g_{it}. We point out that since the objective function of the electricity market-clearing is minimizing social costs—which includes those of reserve provision—, then the most stringent case of the lower bound (C.2) is always binding.

In a similar manner, the down-spinning reserve provided by generator i during period t is determined by the bounds

$$0 \leq r_{it}^{dn} \leq r_{it}^{dn\,\max} u_{it}, \tag{C.3}$$

$$r_{it}^{dn} \geq g_{it} - g_{it}(k, \tau) - g_i^{\max}(2 - u_{it} - u_{it}(k, \tau)), \tag{C.4}$$

for $k = 1, \ldots, K$ and $\tau = 1, \ldots, T$, and where the parameters $r_{it}^{dn\,\max}$ represent the upper limits on the down-spinning reserve offers imposed by each of the generators. Like with up-spinning reserve, down-spinning reserve is set by the largest down-going post-contingency power production deviation away from the pre-contingency dispatch, g_{it}.

C.1.2 Demand-side

In the case of demand at bus m, providing up-spinning reserve during period t boils down to voluntarily reducing its consumption from d_{mt} to $d_{mt}(k, \tau)$, with $(d_{mt}, d_{mt}(k, \tau)) \in \mathcal{D}_{mt}$, given that $m \notin \mathcal{C}_k$—that is, contingency k does not involve a demand disturbance at bus

m. This type of reserve provision must also satisfy the two sets of inequalities

$$0 \leq r_{mt}^{up} \leq r_{mt}^{up\,\max}, \tag{C.5}$$

$$r_{mt}^{up} \geq d_{mt} - d_{mt}(k,\tau), \tag{C.6}$$

for $k = 1,\ldots,K$ and $\tau = 1,\ldots,T$, and where $r_{mt}^{up\,\max}$ is the maximum amount of up-spinning reserve demand at bus m is willing to provide during period t. Similar bounds apply to down-going demand-side spinning reserves if $m \notin \mathcal{C}_k$

$$0 \leq r_{mt}^{dn} \leq r_{mt}^{dn\,\max}, \tag{C.7}$$

and for $k = 1,\ldots,K$ and $\tau = 1,\ldots,T$

$$r_{mt}^{dn} \geq d_{mt}(k,\tau) - d_{mt}, \tag{C.8}$$

where $r_{mt}^{dn\,\max}$ is the maximum down-spinning reserve amount that can be provided at bus m during period t.

C.2 Non-Spinning Reserve

C.2.1 Generation-side

The up-going non-spinning reserve contributions from the generators $i = 1,\ldots,I$ during time periods $t = 1,\ldots,T$ are restricted by

$$0 \leq \tilde{r}_{it}^{up} \leq \tilde{r}_{it}^{up\,\max}(1 - u_{it}), \tag{C.9}$$

and for $\tau = 1,\ldots,T$ and $k = 1,\ldots,K$

$$\tilde{r}_{it}^{up} \geq g_{it}(k,\tau) - g_i^{\max}u_{it}, \tag{C.10}$$

where $\tilde{r}_{it}^{up\,\max}$ are offer-based upper limits on up-going non-spinning reserve. The inequalities (C.9) and (C.10) indicate that up non-spinning reserve can be provided only by generators not already online, which are those with $u_{it} = 0$ and which are capable of being turned on during period t.

The down-going non-spinning reserve supplies from the generators $i = 1, \ldots, I$ and for $t = 1, \ldots, T$ are restricted by

$$0 \leq \tilde{r}_{it}^{dn} \leq \tilde{r}_{it}^{dn\ \max} u_{it}, \tag{C.11}$$

and for $k = 1, \ldots, K$, $\tau = 1, \ldots, T$

$$\tilde{r}_{it}^{dn} \geq g_{it} - g_i^{\max} u_{it}(k, \tau), \tag{C.12}$$

where $\tilde{r}_{it}^{dn\ \max}$ are offer-based upper limits on down-going non-spinning reserve. The inequalities (C.11) and (C.12) indicate that down non-spinning reserve can be provided only by the generators that are already online, that is those with $u_{it} = 1$, which are capable of being shut down during period t.

C.2.2 Demand-side

Demands cannot offer non-spinning reserve services since, under the model developed in this dissertation, they do not have associated binary variables (like u_{it} for the generators) modeling whether they are connected or disconnected from the grid. Here, they are synchronized to the grid by default.

Appendix D

Equivalence of Loss-of-Load Conditions

In Chapter 1 and in [50], it was suggested to use binary variables, which we denote as $\psi_{mt}(k,\tau)$ here, to indicate whether loss-of-load events would occur at bus m during period t if contingency scenario (k,τ) were to happen. These indicator variables take the value 1 if there is loss-of-load, in other words if $l_{mt}(k,\tau) > 0$, or they equal 0 otherwise. These variables and of their corresponding load shedding amounts are calculated for $m = 1,\ldots,M$, $t = 1,\ldots,T$, $k = 1,\ldots,K$, and $t \geq \tau$ using

$$l'_{mt}(k,\tau) = d_{mt}(k,\tau) + \sum_{\substack{\ell\in\mathcal{B}_m \\ \ell\notin\mathcal{C}_k}} f_\ell(\boldsymbol{\delta}_t(k,\tau),k,\tau) - \sum_{\substack{i\in\mathcal{A}_m \\ i\notin\mathcal{C}_k}} g_{it}(k,\tau), \tag{D.1}$$

$$\frac{l'_{mt}(k,\tau)}{Z} \leq \psi_{mt}(k,\tau) \leq 1 + \frac{l'_{mt}(k,\tau)}{Z}, \tag{D.2}$$

$$l_{mt}(k,\tau) = \psi_{mt}(k,\tau) l'_{mt}(k,\tau), \tag{D.3}$$

where $l'_{mt}(k,\tau)$ is a dummy variable, Z is a large positive number and $\psi_{mt}(k,\tau) \in \mathbb{B}$.

Theorem D.1 (Equivalence of loss-of-load conditions). *The conditions on the load shedding variables* (D.1)–(D.3) *are equivalent to those proposed in Chapter 2, which are* (2.8) *and* (2.9) *along with the proposed objective function* (2.11).

Proof. The conditions (D.1)–(D.3) entail (2.8) and (2.9) by inspection.

Next, we show that together (2.8), (2.9) and the cost-minimizing action of (2.11) are sufficient to replace (D.1)–(D.3). We let the optimal amount of load shedding found using (D.1)–(D.3) equal $l^*_{mt}(k,\tau)$ and the one found using (2.8), (2.9) and (2.11) be equal to

$l^{\dagger}_{mt}(k,\tau)$.

As a first test, we assume that $l^{*}_{mt}(k,\tau) < l^{\dagger}_{mt}(k,\tau)$. This assumption commands that $l^{\dagger}_{mt}(k,\tau) > 0$ to satisfy (2.8) and (2.9). However, this constitutes a contradiction of the assumed optimality of $l^{\dagger}_{mt}(k,\tau)$ since $l^{\dagger}_{mt}(k,\tau) = l^{*}_{mt}(k,\tau)$ is also feasible according to (2.8) and (2.9) and generates a lower value of the market-clearing objective.

Next, we let $l^{*}_{mt}(k,\tau) > l^{\dagger}_{mt}(k,\tau)$. This is also a contradiction of the assumed optimality of $l^{*}_{mt}(k,\tau)$ because having $l^{*}_{mt}(k,\tau) = l^{\dagger}_{mt}(k,\tau)$ is feasible too according to (D.1)–(D.3). This lesser amount of load shedding leads to a lower value of the objective function. Therefore, $l^{*}_{mt}(k,\tau) = l^{\dagger}_{mt}(k,\tau)$ must always be satisfied, proving that the two sets of loss-of-load conditions are equivalent for any contingency scenario (k,τ), bus m and time t. □

Appendix E

Test Systems

This appendix lists the characteristics of the test systems used to carry out the case studies. Unless stated otherwise in the main text, the characteristics listed here apply integrally.

E.1 System A: Small-Scale System

System A is the three-bus, three-line, three-generator power system shown in Fig. E.1. The three lossless lines have identical reactances of 0.13 per unit on bases of 41 MVA and 120 kV, maximum power carrying capacities of 55 MVA, mean times to failure of 10 000 hours and forced-outage rates $U_\ell = 5 \times 10^{-4}$ for $\ell = 1, 2, 3$.

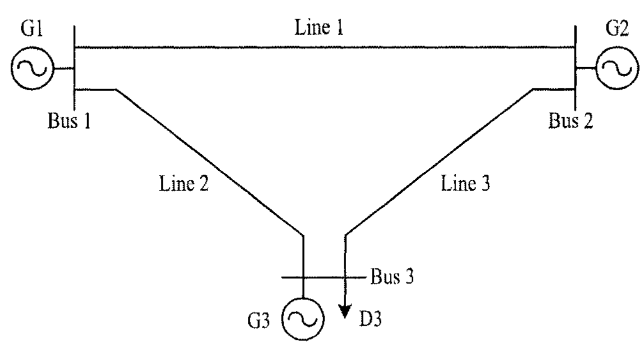

Fig. E.1 Three-bus, three-line, three-generator system—System A

An inelastic demand is located at bus 3 and varies hour by hour according to the pattern detailed in Table E.1. In addition, the demand at bus 3 offers up to 10% of the hourly load as spinning reserve at the rate of \$20 per megawatt-hour for both up- and down-going services. We assume as well that this consumer has a value of lost load (VOLL), v_{3t}, of \$1000 per megawatt-hour applicable during all four hours.

Table E.1 Hourly demand profile at bus 3—System A

	Time t (h)			
	1	2	3	4
d_{3t} (MW)	30	80	110	40

The generating unit data for this system are found in Table E.2. We assume here that the energy and reserve offers of the generators remain unchanged for all hours of the scheduling horizon. Moreover, the generators are assumed not to incur fixed running costs (*i.e.* there are no terms of the form $u_{it}\, c^0_{it}$ inside the market-clearing objective function), but they do incur constant-valued startup costs, c^{su}_i.[1] In this system the generation offering structure requires that each generator offers a single block of energy ranging between its technical minimum, $g^{\min}_i$, and maximum, $g^{\max}_i$, at the rate of a_{it} dollars per megawatt-hour. Moreover, the bounds on the amounts of reserve services offered are set to be the largest possible, in other words, the upper bound on up- and down-spinning reserve is $g^{\max}_i - g^{\min}_i$, and for non-spinning reserves it is equal to $g^{\max}_i$. The generation-side reserve services are offered at rates, in dollars per megawatt-hour, shown in Table E.2 as: q^{up}_{it} for up-going spinning reserve, q^{dn}_{it} for down-going spinning reserve, $\tilde{q}^{up}_{it}$ for up-going non-spinning reserve and $\tilde{q}^{dn}_{it}$ for down-going non-spinning reserve. Finally, the penultimate row of Table E.2 lists the mean times to failure of each of the generators (λ_i^{-1}), while the last row lists the steady-state forced-outage rates of the generators (U_i).

Here, unless it is otherwise stated, the generator minimum up- and down-time constraints are assumed to be inactive, and their ramping capabilities are set to $g^{\max}_i$ megawatts per hour. Finally, we assume that all three generators are in the off state at time $t = 0$.

For the sake of completeness here, we provide in (E.1) the concrete form of the market-

[1] In classical unit commitment models, it is customary to model startup costs of thermal units as being functions of the units' downtime; see [45, 69–71, 75, 77, 79, 80] for more details. Here, however, this is characteristic is not modeled.

Table E.2 Generator data—System A

		Generator i		
		1	2	3
$g_i^{\min}$	(MW)	10.0	10.0	10.0
$g_i^{\max}$	(MW)	100.0	100.0	50.0
a_{it}	(\$/MWh)	30.0	40.0	20.0
c_i^{su}	(\$/h)	100.0	100.0	100.0
q_{it}^{up}	(\$/MWh)	5.0	7.0	8.0
q_{it}^{dn}	(\$/MWh)	5.0	7.0	8.0
$\tilde{q}_{it}^{up}$	(\$/MWh)	4.5	5.5	7.0
$\tilde{q}_{it}^{dn}$	(\$/MWh)	4.5	5.5	7.0
λ_i^{-1}	(h)	500.0	500.0	250.0
U_i	(10^{-3} p.u.)	50.0	20.0	70.0

clearing objective function (2.11) corresponding to this test system.

$$\begin{aligned}\min p(0) \sum_{i=1}^{I} \sum_{t=1}^{T} \left[\tilde{c}_{it}^{su} + a_{it} g_{it} + q_{it}^{up} r_{it}^{up} + q_{it}^{dn} r_{it}^{dn} + \tilde{q}_{it}^{up} \tilde{r}_{it}^{up} + \tilde{q}_{it}^{dn} \tilde{r}_{it}^{dn}\right] \\ + p(0) \sum_{m=1}^{M} \sum_{t=1}^{T} \left[q_{mt}^{up} r_{mt}^{up} + q_{mt}^{dn} r_{mt}^{dn}\right] + \sum_{k=1}^{K} \sum_{\tau=1}^{T} p(k,\tau) \sum_{i=1}^{I} \sum_{t=1}^{T} \left[\tilde{c}_{it}^{su}(k,\tau) + a_{it} g_{it}(k,\tau)\right] \\ + \sum_{m=1}^{M} \sum_{t=1}^{T} v_{mt} \hat{l}_{mt}, \end{aligned} \tag{E.1}$$

where the quantities $\tilde{c}_{it}^{su}$ and $\tilde{c}_{it}^{su}(k,\tau)$ are the startup costs incurred by unit i during period t respectively under the pre- and the different post-contingency scenarios. Note that for generators $i = 1, \ldots, I$, the variables $\tilde{c}_{it}^{su}$ satisfy the following constraints

$$\tilde{c}_{it}^{su} \geq 0; \qquad t = 1, \ldots, T, \tag{E.2}$$

$$\tilde{c}_{it}^{su} \geq c_i^{su}(u_{it} - u_{i(t-1)}); \qquad t = 2, \ldots, T, \tag{E.3}$$

$$\tilde{c}_{it}^{su} \geq c_i^{su}(u_{it} - u_{i0}) \qquad t = 1. \tag{E.4}$$

Likewise, the corresponding post-contingency variables, $\tilde{c}_{it}^{su}(k,\tau)$, should satisfy the constraints (E.2)–(E.4) for all $k = 1, \ldots, K$ and $\tau = 1, \ldots, T$. A corresponding nonanticipation requirement is also needed for all $k = 1, \ldots, K$, $\tau = 1, \ldots, T$, $i = 1, \ldots, I$ and $t < \tau$

$$\tilde{c}^{su}_{it}(k, \tau) = \tilde{c}^{su}_{it}. \tag{E.5}$$

E.2 System B: IEEE Reliability Test System—1996

System B is a larger-scale system based on the single-area version of the IEEE Reliability Test System—1996 [67] shown in Fig. E.2. This is a 24-bus, 38-line and 32-generating unit system used to conduct studies over 24-hour long scheduling horizons.

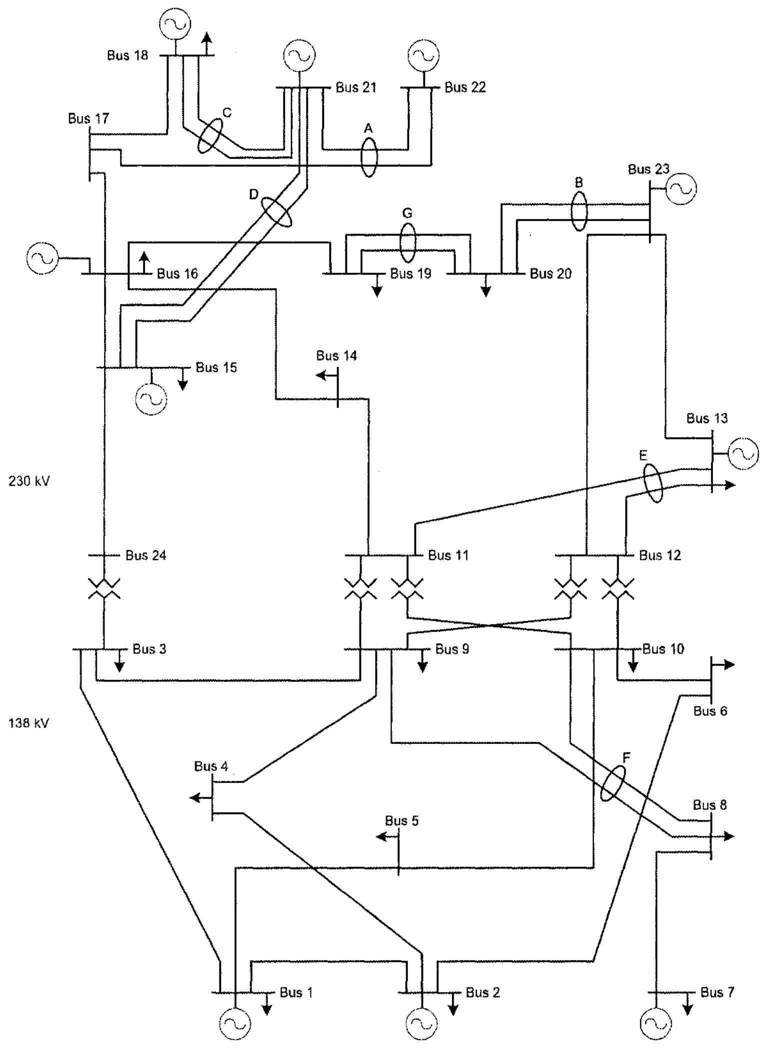

Fig. E.2 Single-area IEEE reliability test system—System B

Here the generating units in the system submit offers to produce energy that consist of four incremental blocks of electrical power output, in kilowatts, versus the unit heat rate, in BTU per kilowatt-hour (as shown in Table 9 of [67]). The assumption here is that the generation offers are identical throughout the scheduling horizon and that they are based on the fuel marginal costs (in dollars per million BTU) found in Table E.3 [194]. The upper limit of the first power output block is considered to be the minimum power output of the thermal generators. In addition, the startup costs are the "cold" values given in Table 8 of [67]. Ramp rates, minimum up- and down-times are as listed in Table 10 of [67]. The up, down, startup and shutdown ramps are assumed to be all equal for a given generating unit. The generator and line failure rate data are found respectively in Tables 6 and 12 of [67]. We note that mean times to failure of the lines $\ell = 1, \ldots, 38$ are estimated from the permanent outage rates and the permanent outage durations (respectively $\lambda_{p\ell}$ and Dur_ℓ in Table 12 of [67]). Thus, for any line ℓ the mean time to failure, in hours, is calculated from

$$\lambda_\ell^{-1} = \frac{8760 - \lambda_{p\ell} Dur_\ell}{\lambda_{p\ell}}, \tag{E.6}$$

where we used the fact that there are 8760 hours in a year.[2] Lastly, we assume that the nuclear (U400) and hydro (U50) generators are must-run units.

Table E.3 Fuel cost data

Fuel type	FO6	FO2	Coal	Nuclear
Cost (\$/MBTU)	2.30	3.00	1.20	0.60

In the tests conducted in this dissertation, the hourly demand data used corresponds to Tuesday of Week 45, Winter Week Day, for the system peak load of 2850 megawatts, while the spatial demand distribution corresponds to that of Table 5 of [67]. The consumers are assumed to be inelastic. In this case, the value of lost load (VOLL) is equal to \$3000 per megawatt-hour for all buses during peak hours—hours 8 to 20 inclusive—while it is equal to \$2000 per megawatt-hour during the remaining off-peak hours. The loads offer spinning reserve, limiting their offers to 2% of their scheduled consumption for all hours and all buses. Both up and down demand-side spinning reserve services are offered at the rate of

[2] Since the permanent outage rates are generally small (order of 0.1 outage per year), it may be reasonable to simply use $\lambda_\ell^{-1} \approx 8760/\lambda_{p\ell}$.

\$50 per megawatt-hour during the entire scheduling horizon.

We assume that the pre- and post-contingency generator on/off variables are set to be equal; therefore, non-spinning reserve services are not available. Moreover, we assume that the generators offer the maximum possible amount of up- and down-going spinning reserve, both at a rate equal to 25% of their highest marginal cost of energy production.

The objective function here is identical to that found in (E.1) except for the following aspects: (i) terms related with non-spinning reserve and post-contingency startup costs are absent; and, (ii) pre- and post-contingency generation costs terms account for the multi-block structure of the offers.

Finally, the initial system conditions (at $t = 0$) are detailed in Table E.4.

Table E.4 Initial operating conditions—System B

Unit i	Type	u_{i0}	g_{i0} (MW)	$UT_{i0}{}^{a}$ (h)	$DT_{i0}{}^{b}$ (h)
1	U12	0	0.0	0	1
2	U12	0	0.0	0	1
3	U12	0	0.0	0	1
4	U12	0	0.0	0	1
5	U12	0	0.0	0	1
6	U20	0	0.0	0	6
7	U20	0	0.0	0	10
8	U20	0	0.0	0	10
9	U20	0	0.0	0	10
10	U50	1	50.0	24	0
11	U50	1	50.0	24	0
12	U50	1	50.0	24	0
13	U50	1	50.0	24	0
14	U50	1	50.0	24	0
15	U50	1	50.0	24	0
16	U76	1	15.2	22	0
17	U76	1	15.2	22	0
18	U76	1	15.2	22	0
19	U76	1	15.2	22	0
20	U100	0	0.0	0	2
21	U100	0	0.0	0	10
22	U100	0	0.0	0	2
23	U155	0	0.0	0	2
24	U155	1	54.3	10	0
25	U155	1	54.3	10	0
26	U155	1	54.3	10	0
27	U197	0	0.0	0	1
28	U197	0	0.0	0	1
29	U197	0	0.0	0	1
30	U350	1	140.0	300	0
31	U400	1	378.1	769	0
32	U400	1	100.0	16	0

[a]Up-time at $t = 0$.
[b]Down-time at $t = 0$.

Appendix F

Computing Tools

F.1 Hardware

Two machines (*Paco* and *Ampère*) were used in solving the electricity market-clearing problems studied in this dissertation. Below is a short description of both.

Paco It is a personal computer equipped with a 1.80-GHz Pentium 4 processor, 512 MB of random-access memory (RAM) and runs under the Microsoft Windows XP Professional operating system.

Ampère It is a Linux-based computation server equipped with eight 1.60-GHz processors and 2 GB of RAM. It is physically located at the Escuela Técnica Superior de Ingenieros Industriales of the Universidad de Castilla-La Mancha, Ciudad Real, Spain.

F.2 Software

The optimization problems developed in this dissertation were coded and solved using the following software packages.

GAMS GAMS, which stands for "General Algebraic Modeling System," is a specialized software package dedicated to formulating mathematical programming problems and interfacing them with third party solution packages [195, 196]. It is commercialized by the GAMS Development Corporation in Washington, DC. Most notably, it provides a powerful high-level language for representing succinctly large and complex systems modeled as optimization problems. The

GAMS language contains all the necessary data constructs required to formulate any kind of mathematical program: *sets*, *variables* (both continuous and discrete), *parameters* and *equations* (to set up constraints and objective functions) [84, 195].

Among other things, GAMS permits the separation of a model from its input data, thus improving the portability of any given model and easing model data sensitivity analyses. Likewise, GAMS permits the complete separation of a model description from its solution algorithm. This feature can prove to be useful when determining computation time benchmarks.

CPLEX CPLEX, a third party solution package being interfaced by GAMS, is a high-performance mixed-integer linear programming (MILP) solver commercialized by ILOG Inc., based in Mountain View, CA [197]. Its mixed-integer solution engine is based on a branch-and-cut algorithm which is complemented with a number of heuristics [57, 81, 83, 84, 124, 198]. CPLEX handles linear programming using either the revised simplex method or an interior-point algorithm [82], as chosen by the user.[1]

Moreover, CPLEX has a powerful pre-processing engine whose role is to eliminate redundancies in models before initiating the branch-and-cut solution process. These reductions can decrease the core memory requirements for given problems, which often lead to corresponding reductions in solution times.

[1] All numerical examples reported in this dissertation used the revised simplex method for linear programming.

Appendix G

Marginal Prices of Energy and Security

Consider the following security-constrained market-clearing problem, minimizing a measure of social cost $W(\mathbf{u}, \mathbf{x})$

$$\min_{\mathbf{u},\mathbf{x}} W(\mathbf{u}, \mathbf{x}) \tag{G.1}$$

subject to

$$\mathbf{H}(\mathbf{u}(0), \mathbf{x}(0), 0) = \mathbf{0} \qquad\qquad (\boldsymbol{\mu}(0)), \tag{G.2}$$

$$\mathbf{H}(\mathbf{u}(k), \mathbf{x}(k), k) = \mathbf{0}; \quad k = 1, \ldots, K \qquad (\boldsymbol{\mu}(k)), \tag{G.3}$$

$$\mathbf{G}(\mathbf{u}, \mathbf{x}) \geq \mathbf{0} \qquad\qquad (\boldsymbol{\sigma}), \tag{G.4}$$

where the vectors $\mathbf{u}$ and $\mathbf{x}$ represent respectively all discrete and continuous variables of the market-clearing problem. The vector constraint (G.2) represents the power balance under the pre-contingency (or error-free) scenario, while (G.3) represents the power balance conditions for each of the pre-selected $k = 1, \ldots, K$ contingency (or net load uncertainty) scenarios. The vector inequality (G.4) gathers all the remaining network and technological constraints. With respect to each of the constraints, (G.2)–(G.4), we define the Lagrange multiplier vectors $\boldsymbol{\mu}(0)$, $\boldsymbol{\mu}(k); k = 1, \ldots, K$ and $\boldsymbol{\sigma}$.

Theorem G.1 (Arroyo-Galiana Theorem on marginal costing of energy and security). *Given an optimal solution* $(\mathbf{u}^*, \mathbf{x}^*)$ *to the security-constrained market-clearing problem* (G.1)–(G.4), *under reasonable smoothness assumptions of the functions* $W(\cdot)$, $\mathbf{H}(\cdot)$ *and* $\mathbf{G}(\cdot)$ *with respect to the continuous variables* $\mathbf{x}$, *and while we keep the discrete variables fixed at* $\mathbf{u}^*$, *the marginal costs of energy and security associated with this stationary point* *are respectively*

$$\frac{\partial W}{\partial \mathbf{E}} = \boldsymbol{\mu}(0) + \sum_{k=1}^{K} \boldsymbol{\mu}(k), \tag{G.5}$$

and

$$\frac{\partial W}{\partial \mathbf{S}} = \sum_{k=1}^{K} \boldsymbol{\mu}(k), \tag{G.6}$$

where $d\mathbf{E}$ *represents an incremental vector perturbation of the power balance relations under the pre-contingency (or error-free) scenario* (G.2) *and* $d\mathbf{S}$ *is an extra incremental perturbation of the power balance relations in the failed (or net load uncertainty-modeling) states* (G.3).

Proof. See [64] for a formal proof. □

Corollary G.1. *Under marginal pricing [3, 125], the marginal costs defined by* (G.5) *and* (G.6) *become respectively the nodal prices of energy and security.*

Appendix H

Discretization of the Net Load Error Probability Distribution

H.1 Net Load Error Probability Calculations

Fig. 4.1 shows an example of a possible discretization pattern of the net load error RV probability distribution function. Taking the midpoints of the intervals to represent the discrete realizations of the net load error RV $\theta_{nt}(j)$, their respective probabilities $\nu_t(j)$, $j = 1, \ldots, J$ are the areas below the probability distribution function evaluated between the endpoints of the intervals. The intervals corresponding to the tails of the distribution, which are those with indices $j = 1$ and $j = J$, must account for the remaining values of the net load error RV which extend to $-\infty$ and to $+\infty$ respectively.

Given that an interval j has an upper endpoint $\theta_{nt}^{ub}(j)$ and a lower endpoint $\theta_{nt}^{lb}(j)$, the associated probability is calculated as

$$\nu_t(j) = \frac{1}{\sqrt{2\pi}\sigma_{nt}} \int_{\theta_{nt}^{lb}(j)}^{\theta_{nt}^{ub}(j)} e^{-\zeta^2/2\sigma_{nt}^2}\, d\zeta = \frac{1}{2}\operatorname{erf}\left(\frac{\theta_{nt}^{ub}(j)}{\sqrt{2}\sigma_{nt}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{\theta_{nt}^{lb}(j)}{\sqrt{2}\sigma_{nt}}\right), \tag{H.1}$$

where erf(·) is known as the "error function" [199]

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\, dt. \tag{H.2}$$

For example, in the case of Fig. 4.1, where $\sigma_{nt} = 0.1$ per unit, the probabilities of the intervals are calculated as

$$\nu_t(1) = \frac{1}{\sqrt{2\pi}\sigma_{nt}} \int_{-\infty}^{-0.25} e^{-\zeta^2/2\sigma_{nt}^2}\, d\zeta = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{-0.25}{\sigma_{nt}\sqrt{2}}\right)\right] = 0.0062,$$

$$\nu_t(2) = \frac{1}{\sqrt{2\pi}\sigma_{nt}} \int_{-0.25}^{-0.15} e^{-\zeta^2/2\sigma_{nt}^2}\, d\zeta = \frac{1}{2}\left[\mathrm{erf}\left(\frac{-0.15}{\sigma_{nt}\sqrt{2}}\right) - \mathrm{erf}\left(\frac{-0.25}{\sigma_{nt}\sqrt{2}}\right)\right] = 0.0606,$$

$$\vdots$$

$$\nu_t(7) = \frac{1}{\sqrt{2\pi}\sigma_{nt}} \int_{0.25}^{\infty} e^{-\zeta^2/2\sigma_{nt}^2}\, d\zeta = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{0.25}{\sigma_{nt}\sqrt{2}}\right)\right] = 0.0062.$$

The full set of probabilities is found in Table H.1. We note here that $\sum_{j=1}^{J} \nu_t(j) = 1$ is satisfied, as it should.

Table H.1 Discrete net load error probabilities

j	1	2	3	4	5	6	7
$\nu_t(j)$	0.0062	0.0606	0.2417	0.3829	0.2417	0.0606	0.0062

H.2 Net Load Error Scenario Probability Calculations

Given a net load error scenario $\mathcal{S}_k = \{j_{k1}, j_{k2}, \ldots, j_{kT}\}$, the associated sequence of node probabilities $p_t(k)$ for $t = 1, \ldots, T$ is calculated recursively from the product

$$p_t(k) = p_{(t-1)}(k)\, \nu_t(j_{kt}), \tag{H.3}$$

where $p_0(k) = 1$. It should be noted that for each time period t the condition $\sum_{k=1}^{K} p_t(k) = 1$ is satisfied, as it should.

References

[1] F. C. Schweppe, "Power systems '2000': Hierarchical control strategies," *IEEE Spectrum*, vol. 15, no. 7, pp. 42–47, July 1978.

[2] F. C. Schweppe, R. D. Tabors, and J. L. Kirtley, "Homeostatic control for electric power usage," *IEEE Spectrum*, vol. 19, no. 7, pp. 44–48, July 1982.

[3] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn, *Spot Pricing of Electricity*. Boston, MA: Kluwer Academic Publishers, 1988.

[4] M. Huneault, F. D. Galiana, and G. Gross, "A review of the restructuring in the electricity business," in *Proc. 13th Power Systems Computation Conference*, Trondheim, Norway, 1999.

[5] (1996) Order no. 888: Promoting wholesale competition through open access non-discriminatory transmission services by public utilities; recovery of stranded costs by public utilities and transmitting utilities. United States Federal Energy Regulatory Commission. Washington, DC. [Online]. Available: http://www.ferc.gov/legal/maj-ord-reg/land-docs/order888.asp.

[6] (1996) Order no. 889: Open access same-time information system (formerly real-time information networks) and standards of conduct. United States Federal Energy Regulatory Commission. Washington, DC. [Online]. Available: http://www.ferc.gov/legal/maj-ord-reg/land-docs/order889.asp.

[7] B. R. Barkovich and D. V. Hawk, "Charting a new course in California," *IEEE Spectrum*, vol. 33, no. 7, pp. 26–31, July 1996.

[8] C. Mensah-Bonsu and S. S. Oren, "California electricity market crisis: Causes, remedies, and prevention," *IEEE Power Eng. Rev.*, vol. 22, no. 8, pp. 4–5, Aug. 2002.

[9] M. D. Ilić and F. D. Galiana, "Power system operation: Old vs. new," in *Power System Restructuring: Engineering and Economics*, M. D. Ilić, F. D. Galiana, and L. H. Fink, Eds. Norwell, MA: Kluwer Academic Publishers, 1998.

[10] S. Stoft, *Power System Economics: Designing Markets for Electricity.* New York, NY: IEEE/Wiley Interscience, 2002.

[11] F. D. Galiana and A. J. Conejo, "Operación del sistema de generación," in *Análisis y Operación de Sistemas de Energía Eléctrica*, A. Gómez Expósito, Ed. Madrid, Spain: McGraw-Hill, 2002.

[12] (2002) Working paper on standardized transmission service and wholesale electric market design. United States Federal Energy Regulatory Commission. Washington, DC. [Online]. Available: http://elibrary.ferc.gov/idmws/nvcommon/NVViewer.asp?Doc=9053848:0.

[13] X. Ma, D. I. Sun, and K. W. Cheung, "Evolution toward standardized market design," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 460–469, May 2003.

[14] P. Joskow and J. Tirole, "Reliability and competitive electricity markets," University of California Energy Institute, Berkeley, CA, Tech. Rep. CESM WP 129, Apr. 2004. [Online]. Available: http://www.ucei.berkeley.edu/PDF/csemwp129.pdf.

[15] Canada-U.S. Power System Outage Task Force, "Final report on the August 14, 2003 blackout in the United States and Canada: Causes and recommendations," Natural Resources Canada, Ottawa, ON, Tech. Rep., 2004. [Online]. Available: http://www.nrcan-rncan.gc.ca/media/docs/final/finalrep_e.htm.

[16] R. Bacher and U. Näf, "Report on the blackout in Italy on 28 September 2003," Swiss Federal Office of Energy, Berne, Switzerland, Tech. Rep., 2003.

[17] "Power failure in Eastern Denmark and Southern Sweden on 23 September 2003: Final report on the course of events," Elkraft System, Ballerup, Denmark, Tech. Rep., 2003.

[18] "Report on support investigations into recent blackouts in London and West Midlands—volume 1 main report," Office of Gas and Electricity Markets, London, United Kingdom, Tech. Rep., 2004.

[19] C. Vournas, "Technical summary on the Athens and Southern Greece blackout of July 12, 2004," Regulatory Authority for Energy, Athens, Greece, Tech. Rep., 2004.

[20] D. G. Luenberger, *Investment Science.* New York, NY: Oxford University Press, 1997.

[21] J. Wang, N. E. Redondo, and F. D. Galiana, "Demand-side reserve offers in joint energy/reserves electricity markets," *IEEE Trans. Power Syst.*, vol. 18, no. 4, pp. 1300–1306, Nov. 2003.

[22] J. J. Shaw, "A direct method for security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 10, no. 3, pp. 1329–1342, Aug. 1995.

[23] T. Alvey, D. Goodwin, X. Ma, D. Streiffert, and D. Sun, "A security-constrained bid-clearing system for the New Zealand wholesale electricity market," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 340–346, May 1998.

[24] M. Aganagic, K. H. Abdul-Rahman, and J. G. Waight, "Spot pricing of capacities for generation and transmission of reserve in an extended poolco model," *IEEE Trans. Power Syst.*, vol. 13, no. 3, pp. 1128–1135, Aug. 1998.

[25] N. S. Rau, "Optimal dispatch of a system based on offers and bids—a mixed-integer LP formulation," *IEEE Trans. Power Syst.*, vol. 14, no. 1, pp. 274–279, Feb. 1999.

[26] A. I. Cohen, V. Brandwajn, and S.-K. Chang, "Security constrained unit commitment for open markets," in *Proc. IEEE PICA*, Santa Clara, CA, 1999, pp. 39–44.

[27] H. Ma and S. M. Shahidehpour, "Unit commitment with transmission security and voltage constraints," *IEEE Trans. Power Syst.*, vol. 14, no. 2, pp. 757–764, May 1999.

[28] K. W. Cheung, P. Shamsollahi, D. Sun, J. Milligan, and M. Potishnak, "Energy and ancillary service dispatch for the interim ISO New England electricity market," *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 968–974, Aug. 2000.

[29] M. Madrigal and V. H. Quintana, "A security-constrained energy and spinning reserve markets clearing system using an interior-point method," *IEEE Trans. Power Syst.*, vol. 15, no. 4, pp. 1410–1416, Nov. 2000.

[30] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management.* New York, NY: IEEE/Wiley Interscience, 2002.

[31] Z. Li and M. Shahidehpour, "Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1079–1088, May 2005.

[32] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. New York, NY: Wiley Interscience, 1996.

[33] T. M. Athay, "Generation scheduling and control," *Proc. IEEE*, vol. 75, no. 12, pp. 1592–1606, Dec. 1987.

[34] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary and tertiary reserves," *Proc. IEEE*, vol. 93, no. 11, pp. 1970–1983, Nov. 2005.

[35] (2005) Glossary of terms used in reliability standards. North American Electric Reliability Council. Princeton, NJ. [Online]. Available: ftp://www.nerc.com/pub/sys/all_updl/standards/sar/Glossary_Clean_1-07-05.pdf.

[36] J. F. Restrepo and F. D. Galiana, "Unit commitment with primary frequency regulation constraints," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1836–1842, Nov. 2005.

[37] B. F. Hobbs, W. R. Stewart Jr., R. E. Bixby, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, "Why this book? New capabilities and new needs for unit commitment modeling," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, Eds. Norwell, MA: Kluwer Academic Publishers, 2001.

[38] L. H. Fink, "Security: Its meaning and objectives," in *Proc. International Symposium on Power System Security Assessment*, Ames, IA, 1988.

[39] (2004) PJM manual 11: Scheduling operations, revision 22. PJM Interconnection, LLC. Valley Forge, PA. [Online]. Available: http://www.pjm.com/contributions/pjm-manuals/pdf/m11v22.pdf.

[40] (1998) Operación del sistema eléctrico, procedimientos de operación. Red Eléctrica de España. Madrid, Spain. [Online]. Available: http://www.ree.es/cap03/pdf/po/PO_resol_30jul1998_b.pdf.

[41] T. E. Dy Liacco, "The adaptive reliability control system," *IEEE Trans. Power App. Syst.*, vol. PAS-86, no. 5, pp. 517–528, May 1967.

[42] D. S. Kirschen, "Power system security," *Power Eng. J.*, vol. 16, no. 5, pp. 241–248, Oct. 2002.

[43] L. T. Anstine, R. E. Burke, J. E. Casey, R. Holgate, R. S. John, and H. G. Stewart, "Application of probability methods to the determination of spinning reserve requirement for the Pennsylvania-New Jersey-Maryland Interconnection," *IEEE Trans. Power App. Syst.*, vol. PAS-82, pp. 726–735, Oct. 1963.

[44] J. D. Guy, "Security constrained unit commitment," *IEEE Trans. Power App. Syst.*, vol. PAS-90, no. 3, pp. 1385–1390, May/June 1971.

[45] T. S. Dillon, K. W. Edwin, H.-D. Kochs, and R. J. Taud, "Integer programming approach to the problem of optimal unit commitment with probabilistic reserve determination," *IEEE Trans. Power App. Syst.*, vol. PAS-97, no. 6, pp. 2154–2166, Nov./Dec. 1978.

[46] R. Billinton and R. N. Allan, *Reliability Evaluation of Power Systems*, 2nd ed. New York, NY: Plenum Press, 1996.

[47] H. B. Gooi, D. P. Mendes, K. R. W. Bell, and D. S. Kirschen, "Optimal scheduling of spinning reserve," *IEEE Trans. Power Syst.*, vol. 14, no. 4, pp. 1485–1492, Nov. 1999.

[48] M. Flynn, W. P. Sheridan, J. D. Dillon, and M. J. O'Malley, "Reliability and reserve competitive electricity market scheduling," *IEEE Trans. Power Syst.*, vol. 16, no. 1, pp. 78–87, Feb. 2001.

[49] D. Chattapadhyay and R. Baldick, "Unit commitment with probabilistic reserve," in *Proc. IEEE Power Eng. Soc. Winter Meeting*, vol. 1, New York, NY, 2002, pp. 280–285.

[50] F. Bouffard and F. D. Galiana, "An electricity market with a probabilistic spinning reserve criterion," *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 300–307, Feb. 2004.

[51] E. Castillo, A. J. Conejo, R. Mínguez, and C. Castillo, "An alternative approach for addressing the failure probability-safety factor method with sensitivity analysis," *Reliab. Eng. Syst. Safe.*, vol. 82, no. 2, pp. 207–216, Nov. 2003.

[52] R. Billinton and R. Mo, "Deterministic/probabilistic contingency evaluation in composite generation and transmission systems," in *Proc. IEEE Power Eng. Soc. Gen. Meeting*, Denver, CO, 2004, pp. 2232–2237.

[53] F. Bouffard, F. D. Galiana, and A. J. Conejo, "Market-clearing with stochastic security—Part I: Formulation," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1818–1826, Nov. 2005.

[54] ——, "Market-clearing with stochastic security—Part II: Case studies," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1827–1835, Nov. 2005.

[55] "Forced outage performance of transmission equipment," Canadian Electricity Association, Ottawa, ON, Tech. Rep., 2003.

[56] "Generation equipment status annual report," Canadian Electricity Association, Ottawa, ON, Tech. Rep., 2003.

[57] C. A. Floudas, *Nonlinear and Mixed-Integer Optimization: Fundamentals and Applications.* New York, NY: Oxford University Press, 1995.

[58] H. Singh and A. Papalexopoulos, "Competitive procurement of ancillary services by an independent system operator," *IEEE Trans. Power Syst.*, vol. 14, no. 2, pp. 498–504, May 1999.

[59] S. S. Oren, "Design of ancillary services markets," in *Proc. 34th Hawaii International Conference on System Sciences*, Maui, HI, 2001.

[60] A. Papalexopoulos and H. Singh, "On the various design options for ancillary services markets," in *Proc. 34th Hawaii International Conference on System Sciences*, Maui, HI, 2001.

[61] G. Strbac and D. S. Kirschen, "Who should pay for reserves?" *Elect. J.*, vol. 13, no. 8, pp. 32–37, Oct. 2000.

[62] B. Kirby and E. Hirst, "Allocating the costs of contingency reserves," *Elect. J.*, vol. 16, no. 10, pp. 39–47, Dec. 2003.

[63] T. Wu, M. Rothleder, Z. Alaywan, and A. D. Papalexopoulos, "Pricing energy and ancillary services in integrated market systems by an optimal power flow," *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 339–347, Feb. 2004.

[64] J. M. Arroyo and F. D. Galiana, "Energy and reserve pricing in security and network-constrained electricity markets," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 634–643, May 2005.

[65] G. M. Masters, *Renewable and Efficient Electric Power Systems.* New York, NY: IEEE/Wiley Interscience, 2004.

[66] T. Ackermann, Ed., *Wind Power in Power Systems.* New York, NY: Wiley, 2005.

[67] Reliability Test System Task Force, "The IEEE reliability test system—1996," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 1010–1020, Aug. 1999.

[68] A. L. Motto, F. D. Galiana, A. J. Conejo, and J. M. Arroyo, "Network-constrained multi-period auction for a pool-based electricity market," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 646–653, Aug. 2002.

[69] A. Turgeon, "Optimal unit commitment," *IEEE Trans. Automat. Contr.*, vol. AC-22, no. 2, pp. 223–227, Apr. 1977.

[70] ——, "Optimal scheduling of thermal generating units," *IEEE Trans. Automat. Contr.*, vol. AC-23, no. 6, pp. 1000–1005, Dec. 1978.

[71] D. P. Bertsekas, G. S. Lauer, N. R. Sandell Jr., and T. A. Posbergh, "Optimal short-term scheduling of large-scale power systems," *IEEE Trans. Automat. Contr.*, vol. AC-28, no. 1, pp. 1–11, Jan. 1983.

[72] A. I. Cohen and M. Yoshimura, "A branch-and-bound algorithm for unit commitment," *IEEE Trans. Power App. Syst.*, vol. PAS-102, no. 2, pp. 444–450, Feb. 1983.

[73] A. I. Cohen and V. R. Sherkat, "Optimization-based methods for operations scheduling," *Proc. IEEE*, vol. 75, no. 12, pp. 1574–1591, Dec. 1987.

[74] S. K. Tong and S. M. Shahidehpour, "An innovative approach to generation scheduling in large-scale hydro-thermal power systems with fuel constrained units," *IEEE Trans. Power Syst.*, vol. 5, no. 2, pp. 665–673, May 1990.

[75] G. B. Sheblé and G. N. Fahd, "Unit commitment literature synopsis," *IEEE Trans. Power Syst.*, vol. 9, no. 1, pp. 128–135, Feb. 1994.

[76] R. Baldick, "Generalized unit commitment," *IEEE Trans. Power Syst.*, vol. 10, no. 1, pp. 465–475, Feb. 1995.

[77] J. M. Arroyo and A. J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Trans. Power Syst.*, vol. 15, no. 3, pp. 1225–1231, Aug. 2000.

[78] ——, "Optimal response of a power generator to energy, AGC, and reserve pool-based markets," *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 404–410, May 2002.

[79] M. C. Ruiz-Peinado, "Coordinación hydrotérmica mediante programación lineal entera-mixta," E.T.S.I. Industriales, Universidad de Castilla-La Mancha, Ciudad Real, Spain, Tech. Rep., 2003.

[80] N. P. Padhy, "Unit commitment—a bibliographical survey," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 1196–1205, May 2004.

[81] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization.* New York, NY: Wiley Interscience, 1988.

[82] D. Bertsimas and J. N. Tsitsiklis, *Introduction to Linear Optimization.* Belmont, MA: Athena Scientific, 1997.

[83] L. A. Wolsey, *Integer Programming.* New York, NY: Wiley Interscience, 1998.

[84] E. Castillo, A. J. Conejo, P. Pedregal, R. García, and N. Aguacil, *Building and Solving Mathematical Programming Models in Engineering and Science.* New York, NY: Wiley Interscience, 2002.

[85] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming.* New York, NY: Springer, 1997.

[86] S. N. Siddiqi and M. L. Baughman, "Reliability differentiated real-time pricing of electricity," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 548–554, May 1993.

[87] ——, "Reliability differentiated pricing of spinning reserve," *IEEE Trans. Power Syst.*, vol. 10, no. 3, pp. 1211–1218, Aug. 1995.

[88] K. K. Kariuki and R. N. Allan, "Evaluation of reliability worth and value of lost load," *IEE Proc.—Gener. Transm. Distrib.*, vol. 143, no. 2, pp. 171–180, Mar. 1996.

[89] G. Strbac, S. Ahmed, D. S. Kirschen, and R. N. Allan, "A method for computing the value of corrective security," *IEEE Trans. Power Syst.*, vol. 13, no. 3, pp. 1096–1102, Aug. 1998.

[90] D. S. Kirschen, K. R. W. Bell, D. P. Nedic, D. Jayaweera, and R. N. Allan, "Computing the value of security," *IEE Proc.—Gener. Transm. Distrib.*, vol. 150, no. 6, pp. 673–678, Nov. 2003.

[91] R. T. Rockafellar and R. J.-B. Wets, "Scenarios and policy aggregation in optimization under uncertainty," *Math. Oper. Res.*, vol. 16, no. 1, pp. 119–147, Feb. 1991.

[92] S. Takriti, J. R. Birge, and E. Long, "A stochastic model for the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1497–1508, Aug. 1996.

[93] M. F. V. Pereira and L. M. V. G. Pinto, "Stochastic optimization of a multi-reservoir hydroelectric system—a decomposition approach," *Water Resour. Res.*, vol. 21, no. 6, pp. 779–792, June 1985.

[94] ——, "Multi-stage stochastic optimization applied to energy planning," *Math. Program.*, vol. 52, no. 1–3, pp. 359–375, May 1991.

[95] D. Dentcheva and W. Römisch, "Optimal power generation under uncertainty via stochastic programming," in *Stochastic Programming Methods and Technical Applications*, K. Marti and P. Kall, Eds. Berlin, Germany: Springer-Verlag, 1998.

[96] R. Gollmer, M. P. Nowak, W. Römisch, and R. Schultz, "Unit commitment in power generation—a basic model and some extensions," *Ann. Oper. Res.*, vol. 96, pp. 167–189, Apr. 2000.

[97] U. A. Ozturk, M. Mazumdar, and B. A. Norman, "A solution to the stochastic unit commitment problem using chance constrained programming," *IEEE Trans. Power Syst.*, vol. 19, no. 3, pp. 1589–1598, Aug. 2004.

[98] E. H. Allen, "Stochastic unit commitment in a deregulated electricity industry," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, 1998.

[99] E. H. Allen and M. D. Ilić, *Price-Based Commitment Decisions in the Electricity Market*. London, United Kingdom: Springer, 1999.

[100] S. Takriti, B. Krasenbrink, and L. S.-Y. Wu, "Incorporating fuel constraints and electricity spot prices into the stochastic unit commitment problem," *Oper. Res.*, vol. 48, no. 2, pp. 268–280, Mar./Apr. 2000.

[101] R. Rajaraman, L. Kirsch, F. L. Alvarado, and C. Clark, "Optimal self-commitment under uncertain energy and reserve prices," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, Eds. Norwell, MA: Kluwer Academic Publishers, 2001.

[102] C.-L. Tseng, "A stochastic model for price-based unit commitment problem and its application to short-term generation asset valuation," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, Eds. Norwell, MA: Kluwer Academic Publishers, 2001.

[103] J. Valenzuela and M. Mazumdar, "Probabilistic unit commitment under a deregulated market," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, Eds. Norwell, MA: Kluwer Academic Publishers, 2001.

[104] A. J. Conejo, F. J. Nogales, J. M. Arroyo, and R. García-Bertrand, "Risk-constrained self-scheduling of a thermal power producer," *IEEE Trans. Power Syst.*, vol. 19, no. 3, pp. 1569–1574, Aug. 2004.

[105] J. Cabrero, A. Baíllo, S. Cerisola, M. Ventosa, A. García-Alcalde, F. Perán, and G. Relaño, "A medium-term integrated risk management model for a hydrothermal generation company," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1379–1388, Aug. 2005.

[106] A. J. Monticelli, M. V. F. Pereira, and S. Granville, "Security-constrained optimal power flow with post-contingency corrective rescheduling," *IEEE Trans. Power Syst.*, vol. 2, no. 1, pp. 175–182, Feb. 1987.

[107] R. J. Kaye, F. F. Wu, and P. Varaiya, "Pricing for system security," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 575–583, May 1995.

[108] L. M. Kimball, K. A. Clements, and P. W. Davis, "An implementation of the stochastic OPF problem," *Electr. Power Compon. Syst.*, vol. 31, no. 12, pp. 1193–1204, Dec. 2003.

[109] J. Wang, X. Wang, and Y. Wu, "Operating reserve model in the power market," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 223–229, Feb. 2005.

[110] A. J. Conejo, F. D. Galiana, J. M. Arroyo, R. García-Bertrand, C. W. Chua, and M. Huneault, "Economic inefficiencies and cross-subsidies in an auction-based electricity pool," *IEEE Trans. Power Syst.*, vol. 18, no. 1, pp. 221–228, Feb. 2003.

[111] M. Rashidinejad, Y. H. Song, and M. H. Javidi Dasht-Bayaz, "Contingency reserve pricing via a joint energy and reserve dispatching approach," *Energ. Convers. Manage.*, vol. 43, no. 4, pp. 537–548, Mar. 2002.

[112] P. Carpentier, G. Cohen, J.-C. Culioli, and A. Renaud, "Stochastic optimization of unit commitment: A new decomposition framework," *IEEE Trans. Power Syst.*, vol. 11, no. 2, pp. 1067–1073, May 1996.

[113] G. Wacker and R. Billinton, "Customer cost of electric service interruptions," *Proc. IEEE*, vol. 77, no. 6, pp. 919–930, June 1989.

[114] M. J. Sullivan, T. Vardell, B. N. Suddeth, and A. Vojdani, "Interruption costs, customer satisfaction and expectations for service reliability," *IEEE Trans. Power Syst.*, vol. 11, no. 2, pp. 989–995, May 1996.

[115] J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," *Numerische Mathematik*, vol. 4, pp. 238–252, 1962.

[116] J. R. Birge, "Current trends in stochastic programming computation and applications," Dept. of Industrial and Operations Engineering, Univ. of Michigan, Ann Arbor, MI, Tech. Rep., 1995.

[117] N. Alguacil and A. J. Conejo, "Multiperiod optimal power flow using Benders decomposition," *IEEE Trans. Power Syst.*, vol. 15, no. 1, pp. 196–201, Feb. 2000.

[118] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsbergh, and P. H. Vance, "Branch-and-price: Column generation for solving huge integer programs," *Oper. Res.*, vol. 46, no. 3, pp. 316–329, May/June 1998.

[119] F. Vanderbeck, "On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm," *Oper. Res.*, vol. 48, no. 1, pp. 111–128, Jan./Feb. 2000.

[120] G. Lulli and S. Sen, "A branch-and-price algorithm for multistage stochastic integer programming with application to stochastic batch-sizing problems," *Manage. Sci.*, vol. 50, no. 6, pp. 786–796, June 2004.

[121] J. Dupačová, N. Gröwe-Kuska, and W. Römisch, "Scenario reduction in stochastic programming: An approach using probability metrics," *Math. Program., Ser. A*, vol. 95, no. 3, pp. 493–511, Mar. 2003.

[122] H. Heitsch and W. Römisch, "Scenario reduction algorithms in stochastic programming," *Comput. Optim. Appl.*, vol. 24, no. 2–3, pp. 187–206, Feb./Mar. 2003.

[123] N. Gröwe-Kuska, H. Heitsch, and W. Römisch, "Scenario reduction and scenario tree construction for power management problems," in *Proc. IEEE PowerTech*, vol. 3, Bologna, Italy, 2003.

[124] R. E. Bixby, M. Fenelon, Z. Gu, E. Rothberg, and R. Wunderling, "MIP: Theory and practice—closing the gap," in *System Modeling and Optimization: Methods, Theory and Applications*, M. J. D. Powell and S. Scholtes, Eds. Norwell, MA: Kluwer Academic Publishers, 2000.

[125] A. Mas-Colell, M. D. Whinston, and J. R. Green, *Microeconomic Theory.* New York, NY: Oxford University Press, 1995.

[126] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed. Belmont, MA: Athena Scientific, 1999.

[127] A. L. Motto and F. D. Galiana, "Equilibrium of auction markets with unit commitment: The need for augmented pricing," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 798–805, Aug. 2002.

[128] F. D. Galiana, A. L. Motto, and F. Bouffard, "Reconciling social welfare, agent profits, and consumer payments in electricity pools," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 452–459, May 2003.

[129] F. Bouffard and F. D. Galiana, "Generalized uplifts in pool-based electricity markets: Coordinating time-dynamic markets," in *Analysis, Control and Optimization of Complex Dynamic Systems*, E.-K. Boukas and R. P. Malhamé, Eds. New York, NY: Springer, 2005.

[130] Y. Ren and F. D. Galiana, "Pay-as-bid versus marginal pricing—Part I: Strategic generator offers," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 1771–1776, Nov. 2004.

[131] ——, "Pay-as-bid versus marginal pricing—Part II: Market behavior under strategic generator offers," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 1777–1783, Nov. 2004.

[132] "NETA—The next phase," ILEX Energy Consulting Limited, Oxford, United Kingdom, Tech. Rep., 2002. [Online]. Available: http://www.ilex.co.uk/pages/NetaTheNextPhase26Mar02.pdf.

[133] H. R. Mashhadi, J. Sadeh, and M. A. Latifi, "Optimal supply bidding with risk management in an electricity pay-as-bid auction," in *Proc. 15th Power Systems Computation Conference*, Liège, Belgium, 2005.

[134] A. L. Ott, "Experience with PJM market operation, system design, and implementation," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 528–534, May 2003.

[135] S. Hao and F. Zhuang, "New models for integrated short-term forward electricity markets," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 478–485, May 2003.

[136] F. Ding and J. D. Fuller, "Nodal, uniform, or zonal pricing: Distribution of economic surplus," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 875–882, May 2005.

[137] L. M. Xia, H. B. Gooi, and J. Bai, "A probabilistic reserve with zero-sum settlement scheme," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 993–1000, May 2005.

[138] "Moving forward on climate change: A plan for honouring our Kyoto commitment," Government of Canada, Tech. Rep., 2005. [Online]. Available: http://www.climatechange.gc.ca/kyoto_commitments/report_e.pdf.

[139] "World energy use and carbon dioxide emissions, 1980–2001," United States of America Department of Energy, Energy Information Administration, Washington, DC, Tech. Rep., 2004. [Online]. Available: http://www.eia.doe.gov/emeu/cabs/carbonemiss/energycarbon2004.pdf.

[140] "Summary of Canada's 2003 greenhouse gas inventory," Environment Canada, Ottawa, ON, Tech. Rep., 2005. [Online]. Available: http://www.ec.gc.ca/pdb/ghg/2005summary/2005summary_e.cfm.

[141] H. Holttinen, "Hourly wind power variation in the nordic countries," *Wind Energy*, vol. 8, no. 2, pp. 173–195, Apr./June 2005.

[142] G. Papaefthymiou, P. H. Schavemaker, L. van der Sluis, W. L. Kling, D. Kurowicka, and R. M. Cooke, "Integration of stochastic generation in power systems," in *Proc. 15th Power Systems Computation Conference*, Liège, Belgium, 2005.

[143] E. Hirst, "Integrating wind output with bulk power operations and wholesale electricity markets," *Wind Energy*, vol. 5, no. 1, pp. 19–36, Jan./Mar. 2002.

[144] S. Persaud, B. Fox, and D. Flynn, "Effects of large scale wind power on total system variability an operation: Case study of Northern Ireland," *Wind Eng.*, vol. 27, no. 1, pp. 3–20, Feb. 2003.

[145] B. Parsons, M. Milligan, B. Zavadil, D. Brooks, B. Kirby, K. Dragoon, and J. Caldwell, "Grid impact of wind power: A summary of recent studies in the United States," *Wind Energy*, vol. 7, no. 2, pp. 87–108, Apr./June 2004.

[146] H. Holttinen, "Impact of hourly wind power variations on the system operation in the nordic countries," *Wind Energy*, vol. 8, no. 2, pp. 197–218, Apr./June 2005.

[147] A. Fabbri, T. Gómez San Román, J. Rivier Abbad, and V. H. Méndez Quezada, "Assessment of the cost associated with wind generation prediction errors in a liberalized electricity market," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1440–1446, Aug. 2005.

[148] S. T. Lee and Z. A. Yamayee, "Load-following and spinning-reserve penalties for intermittent generation," *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 3, pp. 1203–1211, Mar. 1981.

[149] R. A. Schlueter, G. L. Park, M. Lotfalian, H. Shayanfar, and J. Dorsey, "Modification of power system operation for significant wind generation penetration," *IEEE Trans. Power App. Syst.*, vol. PAS-102, no. 1, pp. 153–161, Jan. 1983.

[150] S. M. Chan, D. C. Powell, M. Yoshimura, and D. H. Curtice, "Operations requirements of utilities with wind power generation," *IEEE Trans. Power App. Syst.*, vol. PAS-102, no. 9, pp. 2850–2860, Sept. 1983.

[151] L. Söder, "Benefit assessment of wind power in hydrothermal power systems," Ph.D. dissertation, Royal Institute of Technology, Stockholm, Sweden, 1988.

[152] ——, "Reserve margin planning in a wind-hydro-thermal power system," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 564–571, May 1993.

[153] G. Dany, "Power reserve in interconnected systems with high wind power production," in *Proc. IEEE PowerTech*, Porto, Portugal, 2001.

[154] M. R. Milligan, "A chronological reliability model to assess operating reserve allocation to wind power plants," in *Proc. European Wind Energy Conference*, Copenhagen, Denmark, 2001.

[155] M. Korpås, R. Hildrum, and A. T. Holen, "Operation and sizing of energy storage for wind power plant in a market system," in *Proc. 14th Power Systems Computation Conference*, Sevilla, Spain, 2002.

[156] R. Doherty and M. O'Malley, "Quantifying reserve demands due to increasing wind power penetration," in *Proc. IEEE PowerTech*, Bologna, Italy, 2003.

[157] M. O'Malley, G. Strbac, B. Fox, D. Flynn, R. Slark, S. Rourke, and J. Sinner, "Operating reserve requirements as wind power penetration increases in the Irish electricity system," Sustainable Energy Ireland, Dublin, Ireland, Tech. Rep., 2004. [Online]. Available: http://www.irish-energy.ie/uploads/documents/upload/publications/Ilex-Wind-Reser_rev2FSFinal.pdf.

[158] R. Doherty and M. O'Malley, "A new approach to quantify reserve demand in systems with significant installed wind capacity," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 587–595, May 2005.

[159] P. E. Morthorst, "Wind power and the conditions at a liberalized power market," *Wind Energy*, vol. 6, no. 3, pp. 297–308, July/Sept. 2003.

[160] T. Molinski, "Manitoba Hydro work on wind," presentation slides, Manitoba Hydro, McGill University, Montreal, QC, Apr. 2005.

[161] L. Landberg, G. Giebel, H. A. Nielsen, T. Nielsen, and H. Madsen, "Short-term prediction—an overview," *Wind Energy*, vol. 6, no. 3, pp. 273–280, July/Sept. 2003.

[162] C. Abbey, "A doubly-fed induction generator and energy storage system for wind power applications," Master's thesis, McGill University, Montreal, QC, 2004.

[163] M. Gatrell, "Electricity storage for support of distributed and renewable power," presentation slides, National Research Council of Canada, McGill University, Montreal, QC, Nov. 2004.

[164] B. Roberts and J. McDowall, "Commercial successes in power storage," *IEEE Power Energy Mag.*, vol. 3, no. 2, pp. 24–30, Mar./Apr. 2005.

[165] J. V. Paatero and P. D. Lund, "Effect of energy storage on variations in wind power," *Wind Energy*, 2005, in press.

[166] E. Handschin, F. Neise, H. Neumann, and R. Schultz, "Optimal operation of dispersed generation under uncertainty using mathematical programming," in *Proc. 15th Power Systems Computation Conference*, Liège, Belgium, 2005.

[167] G. Gross and F. D. Galiana, "Short-term load forecasting," *Proc. IEEE*, vol. 75, no. 12, pp. 1558–1573, Dec. 1987.

[168] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed. Boston, MA: McGraw-Hill, 1991.

[169] R. E. Bellman, *Dynamic Programmming.* Princeton, NJ: Princeton University Press, 1957.

[170] D. P. Bertsekas, *Dynamic Programming and Optimal Control, Volume 1*, 2nd ed. Belmont, MA: Athena Scientific, 2000.

[171] D. L. Keefer and S. E. Bodily, "Three-point approximations for continuous random variables," *Manage. Sci.*, vol. 29, no. 5, pp. 595–609, May 1983.

[172] D. L. Keefer, "Certainty equivalent for three-point discrete-distribution approximations," *Manage. Sci.*, vol. 40, no. 6, pp. 760–773, June 1994.

[173] J. C. Hull and A. White, "One-factor interest-rate models and the valuation of interest-rate derivative securities," *J. Financ. Quant. Anal.*, vol. 28, no. 2, pp. 235–254, June 1993.

[174] ——, "Numerical procedures for implementing term structure models II: Two-factor models," *J. Derivatives*, vol. 2, pp. 37–49, 1994.

[175] J. Dupačová, G. Consigli, and S. W. Wallace, "Scenarios for multistage stochastic programs," *Ann. Oper. Res.*, vol. 100, pp. 25–53, Dec. 2000.

[176] B. Lu and M. Shahidehpour, "Short-term scheduling of battery in a grid-connected PV/battery system," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1053–1061, May 2005.

[177] C. Wang and S. M. Shahidehpour, "Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1539–1545, Aug. 1994.

[178] ——, "Optimal generation scheduling with ramping costs," *IEEE Trans. Power Syst.*, vol. 10, no. 1, pp. 60–67, Feb. 1995.

[179] C. G. Ejebe and B. F. Wollenberg, "Automatic contingency selection," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 1, pp. 97–109, Jan. 1979.

[180] A. P. Sakis Meliopoulos, C. S. Cheng, and F. Xia, "Performance evaluation of static security analysis methods," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1441–1449, Aug. 1994.

[181] V. Ajjarapu and B. Lee, "Bibliography on voltage stability," *IEEE Trans. Power Syst.*, vol. 13, no. 1, pp. 115–125, Feb. 1998.

[182] A. B. R. Kumar, V. Brandwajn, A. Ipakchi, and R. Adapa, "Integrated framework for dynamic security analysis," *IEEE Trans. Power Syst.*, vol. 13, no. 3, pp. 816–821, Aug. 1998.

[183] C. Fu and A. Bose, "Contingency ranking based on severity indices in dynamic severity analysis," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 980–986, Aug. 1999.

[184] D. Ernst, D. Ruiz-Vega, M. Pavella, P. M. Hirsch, and D. Sobajic, "A unified approach to transient stability contingency filtering, ranking and assessment," *IEEE Trans. Power Syst.*, vol. 16, no. 3, pp. 435–443, Aug. 2001.

[185] V. Brandwajn and M. G. Lauby, "Complete bounding for AC contingency analysis," *IEEE Trans. Power Syst.*, vol. 4, no. 2, pp. 724–729, May 1989.

[186] R. J. Marceau, R. Mailhot, and F. D. Galiana, "A generalized shell for dynamic security analysis in operations planning," *IEEE Trans. Power Syst.*, vol. 8, no. 3, pp. 1098–1106, Aug. 1993.

[187] E. Hnyilicza, S. T. Y. Lee, and F. C. Schweppe, "Steady state security regions: Set-theoretic approach," in *Proc. IEEE PICA*, New Orleans, LA, 1975, pp. 347–355.

[188] R. Fischl, G. C. Ejebe, and J. A. De Maio, "Identification of power system steady-state security regions under load uncertainty," in *Proc. IEEE PES Summer Meeting*, Portland, OR, 1976.

[189] R. Fischl, T. F. Halpin, and A. Guvenis, "The application of decision-theory to contingency selection," *IEEE Trans. Circuits Syst.*, vol. CAS-29, no. 11, pp. 712–723, Nov. 1982.

[190] M. H. Banakar and F. D. Galiana, "Power system security corridors concept and computation," *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 11, pp. 4524–4532, Nov. 1981.

[191] B. Stott, O. Alsaç, and A. J. Monticelli, "Security analysis and optimization," *Proc. IEEE*, vol. 75, no. 12, pp. 1623–1644, Dec. 1987.

[192] F. Bouffard, F. D. Galiana, and J. M. Arroyo, "Umbrella contingencies in security-constrained optimal power flow," in *Proc. 15th Power Systems Computation Conference*, Liège, Belgium, 2005.

[193] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value-at-risk," *J. Risk*, vol. 2, no. 3, pp. 21–41, 2000.

[194] R. Billinton and W. Li, *Reliability Assessment of Electric Power Systems Using Monte Carlo Methods.* New York, NY: Plenum Press, 1994.

[195] A. Brooke, D. Kendrick, A. Meeraus, and R. Raman, *GAMS: A Users Guide.* Washington, DC: GAMS Development Corporation, 2003.

[196] *GAMS: The Solver Manuals.* Washington, DC: GAMS Development Corporation, 2005.

[197] (2005) CPLEX mixed integer optimizer. ILOG, Inc. Mountain View, CA. [Online]. Available: http://www.ilog.com/products/cplex/product/mip.cfm.

[198] S. Ceria, "Solving hard mixed-integer programs for electricity generation," in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. p. Chao, Eds. Norwell, MA: Kluwer Academic Publishers, 2001.

[199] E. W. Weisstein. (2005) Erf. MathWorld—A Wolfram Web Resource. [Online]. Available: http://mathworld.wolfram.com/Erf.html.